



PUTNAM EXAM SEMINAR  
FALL 2010

AUGUST 30

**Exercise 1.** Consider the following game. You are given a pile of at least two marbles and allowed to divide that pile into two smaller (nonempty) piles of sizes  $a$  and  $b$ . You earn exactly  $ab$  points for this move. You perform the same type of move on any of the smaller piles that have at least two marbles, and earn points for those moves in the same way. You continue dividing the resulting piles and earning points until every pile has only a single marble in it, at which point the game ends. In terms of the number of marbles you begin with, what is the highest score you can possibly achieve?