Putnam Exam Seminar — Handout

Fall 2010 — September 27

Exercise 1. Define a sequence $\{a_n\}$ recursively by setting $a_0 = 1$, $a_1 = 2010$ and

$$a_{n+1} = Ba_n - a_{n-1}$$

for $n \geq 1$. Determine the value of B so that

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 3 + 2\sqrt{2}.$$

Exercise 2. Determine the exact value of $\{x_n\}$ if $x_0 = 0$, $x_1 = 1$ and $x_{n+1} = 2x_n + x_{n-1}$ for $n \geq 1$.

Exercise 3. Determine if the series

$$1 + \frac{1}{2}\frac{19}{7} + \frac{2!}{3^2}\left(\frac{19}{7}\right)^2 + \frac{3!}{4^3}\left(\frac{19}{7}\right)^3 + \frac{4!}{5^4}\left(\frac{19}{7}\right)^4 + \cdots$$

converges or diverges.

Exercise 4. Show that the series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \cdots,$$

whose terms are the reciprocals of the integers that have no prime factors larger than 3, converges, and determine its sum.