Putnam Exam Seminar
Fall 2010

Assignment 10
Due November 22

Exercise 1. Let G be a group with identity e and $\phi : G \to G$ a function such that

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

whenever $g_1 g_2 g_3 = e = h_1 h_2 h_3$. Prove that there exists an element $a \in G$ such that $\psi(x) = a\phi(x)$ is a homomorphism (i.e. $\psi(xy) = \psi(x)\psi(y)$ for all $x, y \in G$). [Putnam Exam, 1997, A4]

Exercise 2. Is there a finite abelian group G such that the product of the orders of all its elements is 2^{2009}? [Putnam Exam, 2009, A5]