Putnam Exam Seminar — Assignment 5

Fall 2010 — Due October 11

Exercise 1. Determine whether or not the matrix

$$\begin{pmatrix} 117 & 218 & 344 & 511 & 1007 \\ 101 & 800 & 911 & 578 & 113 \\ 1212 & 14 & 4216 & 178 & 2013 \\ 516 & 19 & 2114 & 104 & 3416 \\ 789 & 534 & 114 & 472 & 300 \end{pmatrix}$$

has an inverse.

Exercise 2. Determine the number of pairs of positive integers (m, n) that satisfy the equation $19m + 102 + 8n = 2010$.

Exercise 3. Consider the set $\{2, 5, 13\}$. Show that if $D \notin \{2, 5, 13\}$ then there exist $A, B \in \{2, 5, 13, D\}$ so that $AB - 1$ is not a perfect square.

Exercise 4. Let A denote the sum of the decimal digits of 4444^{4444} and let B be the sum of the decimal digits of A. Find the sum of the decimal digits of B.

Exercise 5. Prove that every positive integer has a multiple whose decimal representation includes all ten digits.

Exercise 6. Suppose p is an odd prime. Prove that

$$\sum_{j=0}^{p} \binom{p}{j} \binom{p+j}{j} \equiv 2^p + 1 \pmod{p^2}.$$

[Putnam Exam 1991, B-4]