Putnam Exam Seminar
Fall 2010

Assignment 6
Due October 18

Exercise 1. Find all real-valued continuously differentiable functions f defined on the real line such that for all x,

$$(f(x))^2 = 1990 + \int_0^x \left[(f(t))^2 + (f'(t))^2\right]\, dt.$$

[Putnam Exam, 1990, B1]

Exercise 2. Evaluate

$$\int_0^a \int_0^b e^{\max\{b^2x^2, a^2y^2\}}\, dx\, dy$$

where a and b are positive. [Putnam Exam, 1989, A2]

Exercise 3. Evaluate

$$\int_0^{2\pi} \frac{dx}{1+e^{\sin x}}\, dx.$$

Exercise 4. For what pairs (a, b) of positive real numbers does the improper integral

$$\int_b^\infty \left(\sqrt{\sqrt{x+a}-\sqrt{x}} - \sqrt{\sqrt{x}-\sqrt{x-b}}\right) dx$$

converge? [Putnam Exam, 1995, A2]

Exercise 5. Evaluate

$$\int_0^1 \frac{\ln(x+1)}{x^2+1}\, dx.$$

[Putnam Exam, 2005, A5].

Exercise 6. For each continuous function $f : [0,1] \to \mathbb{R}$, let $I(f) = \int_0^1 f(x)\, dx$ and $J(x) = \int_0^1 x(f(x))^2\, dx$. Find the maximum value of $I(f) - J(f)$ over all such functions f. [Putnam Exam, 2006, B5]