PUTNAM EXAM SEMINAR
FALL 2010

ASSIGNMENT 8
DUE NOVEMBER 1

Exercise 1. Find all pairs of real numbers (x, y) satisfying the system of equations

$$\begin{aligned} \frac{1}{x} + \frac{1}{2y} &= (x^2 + 3y^2)(3x^2 + y^2) \\ \frac{1}{x} - \frac{1}{2y} &= 2(y^4 - x^4). \end{aligned}$$

[Putnam Exam, 2001, B-2]

Exercise 2. Assume that x, y and z are all positive real numbers that satisfy the system of equations

$$\begin{aligned} x + y + xy &= 8 \\ y + z + yz &= 15 \\ z + x + xz &= 35. \end{aligned}$$

Determine the value of $x + y + z + xyz$.

Exercise 3. Find all quadruples of real numbers (x_1, x_2, x_3, x_4) such that the sum of any one and the product of the other three is equal to 2.

Exercise 4. Prove that there are only a finite number of possibilities for the ordered triple $T = (x - y, y - z, z - x)$, where x, y and z are complex numbers satisfying the simultaneous equations

$$x(x-1) + 2yz = y(y-1) + 2zx = z(z-1) + 2xy,$$

and list all such triples T. [Putnam Exam, 1986, B-2]

Exercise 5. Find all positive integers $n, k_1, \ldots k_n$ such that

$$\begin{aligned} k_1 + \cdots k_n &= 5n - 4 \\ \frac{1}{k_1} + \cdots \frac{1}{k_n} &= 1. \end{aligned}$$

[Putnam Exam, 2005, B-2]