Putnam Exam Seminar
Fall 2010

Quiz 3
September 22

Problem 1. Given any thirteen real numbers, prove that there are two of them, say x and y, so that

$$0 < \frac{x-y}{1+xy} < \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}.$$

Problem 2. Prove that if any five points are chosen on a sphere, then four of them lie on some closed hemisphere.

Problem 3. Let $M = \{1, 2, 3, \ldots, 2048\}$ and let $X \subseteq M$ such that $|X| = 15$. Show that there are two disjoint subsets of X whose sum of elements is the same. That is, show that we can find $A, B \subseteq X$ with $A \cap B = \emptyset$ and $\sum_{a \in A} a = \sum_{b \in B} b$. If $|X| = 12$ instead is this result still true?

Problem 4. Given a set of $n+1$ positive integers, none of which is greater than $2n$, prove that at least one member of this set must divide another member of this set.

Problem 5. Given a set of $n+1$ positive integers, none of which is greater than $2n$, prove that at least two of the integers are relatively prime.