PUTNAM EXAM SEMINAR
FALL 2010

QUIZ 4
OCTOBER 11

Problem 1. Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)}+\sqrt{\ln(x+3)}}\,dx.$$

[Putnam Exam, 1987, B1]

Problem 2. Find the volume of the region of points (x, y, z) such that

$$(x^2+y^2+z^2+8)^2 \le 36(x^2+y^2).$$

[Putnam Exam, 2006, A1]

Problem 3. Let A be the area of the region in the first quadrant bounded by the line $y = \frac{1}{2}x$, the x-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$. [Putnam Exam, 1994, A2]

Problem 4. Evaluate

$$\int_0^\infty \left(x - \frac{x^3}{2} + \frac{x^5}{2\cdot 4} - \frac{x^7}{2\cdot 4\cdot 6} + \cdots\right)\left(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2\cdot 4^2} + \frac{x^6}{2^2\cdot 4^2\cdot 6^2} + \cdots\right)\,dx.$$

[Putnam Exam, 1997, A3]

Problem 5. Let $p(x) = 2 + 4x + 3x^2 + 5x^3 + 3x^4 + 4x^5 + 2x^6$. For k with $0 < k < 5$ define

$$I_k = \int_0^\infty \frac{x^k}{p(x)}\,dx.$$

For which k is I_k smallest?