PUTNAM EXAM SEMINAR
FALL 2010

QUIZ 6
OCTOBER 25

Problem 1. Find all ordered pairs of real numbers (x, y) which satisfy the system of equations

$$\begin{aligned} \sqrt{x} + \sqrt{y} &= 3 \\ 3x + 2y &= 14. \end{aligned}$$

Problem 2. Curves A, B, C and D are defined in the plane as follows:

$$\begin{aligned} A &= \left\{ (x, y) \,:\, x^2 - y^2 = \frac{x}{x^2 + y^2} \right\}, \\ B &= \left\{ (x, y) \,:\, 2xy + \frac{y}{x^2 + y^2} = 3 \right\}, \\ C &= \left\{ (x, y) \,:\, x^3 - 3xy^2 + 3y = 1 \right\}, \\ D &= \left\{ (x, y) \,:\, 3x^2 y - 3x - y^3 = 0 \right\}. \end{aligned}$$

Prove that $A \cap B = C \cap D$. [Putnam Exam, 1987, A-1]

Problem 3. Show that there is a unique pair of real numbers (x, y) that satisfy the equation

$$(4x^2 + 6x + 4)(4y^2 - 12y + 25) = 28.$$

Problem 4. Find all values of α for which the curves $y = \alpha x^2 + \alpha x + \frac{1}{24}$ and $x = \alpha y^2 + \alpha y + \frac{1}{24}$ are tangent to each other. [Putnam Exam, 2007, A-1]