Putnam Exam Seminar | Quiz 7
Fall 2010 | November 1

Problem 1. Suppose $p(x)$ is a polynomial of degree seven such that $(x-1)^4$ is a factor of $p(x)+1$ and $(x+1)^4$ is a factor of $p(x)-1$. Find $p(x)$.

Problem 2. Let k be a fixed positive integer. The n-th derivative of $\dfrac{1}{x^k-1}$ has the form $\dfrac{P_n(x)}{(x^k-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$. [Putnam Exam, 2002, A1]

Problem 3. Consider all lines which meet the graph of

$$y = 2x^4 + 7x^3 + 3x - 5$$

in four different points, sat (x_i, y_i) for $i = 1, 2, 3, 4$. Show that $\dfrac{x_1+x_2+x_3+x_4}{4}$ is independent of the line and find its value.

Problem 4. Let k be the smallest positive integer with the property that there are distinct integers m_1, m_2, m_3, m_4, m_5 such that the polynomial

$$p(x) = (x-m_1)(x-m_2)(x-m_3)(x-m_4)(x-m_5)$$

has exactly k nonzero coefficients. Find, with proof, a set of integers m_1, m_2, m_3, m_4, m_5 for which this minimum k is achieved. [Putnam Exam, 1985, B1]

Problem 5. Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a. (*Note:* $\lfloor \nu \rfloor$ is the greatest integer less than or equal to ν.) [Putnam Exam, 2005, B1]