



PUTNAM EXAM SEMINAR
FALL 2010

QUIZ 8
NOVEMBER 10

Problem 1. Find the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers x satisfying $x^4 + 36 \leq 13x^2$. [Putnam Exam, 1986, A1]

Problem 2. Determine the minimum value of

$$(r - 1)^2 + \left(\frac{s}{r} - 1\right)^2 + \left(\frac{t}{s} - 1\right)^2 + \left(\frac{4}{t} - 1\right)^2$$

if r , s and t are real numbers with $1 \leq r \leq s \leq t \leq 4$.

Problem 3. For $x > 0$, find the minimum value of

$$\frac{(x + 1/x)^6 - (x^6 + 1/x^6) - 2}{(x + 1/x)^3 + (x^3 + 1/x^3)}.$$

[Putnam Exam, 1998, B1]

Problem 4. Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle. [Putnam Exam, 1996, A1]

Problem 5. Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers x . [Putnam Exam, 2003, A3]