Putnam Exam Seminar — Quiz 9

Fall 2010 — November 15

Problem 1. Consider a set S with a binary operation $*$, that is, for each $a, b \in S$, $a * b \in S$. Assume that $(a * b) * a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$. [Putnam Exam, 2001, A1]

Problem 2. Let S be a non-empty set with an associative operation that is left and right cancellative ($xy = xz$ implies $y = z$, and $yx = zx$ implies $y = z$). Assume that for every a in S the set $\{a^n : n = 1, 2, 3, \ldots\}$ is finite. Must S be a group? [Putnam Exam, 1989, B2]

Problem 3. Let S be a set of real numbers which is closed under multiplication (that is, if a and b are in S, then so is ab). Let T and U be disjoint subsets of S whose union is S. Given that the product of any *three* (not necessarily distinct) elements of T is in T and that the product of any three elements of U is in U, show that at least one of the two subsets T, U is closed under multiplication. [Putnam Exam, 1995, A1]