Math 1311-1 Fall 2006

Calculus I

Third Midterm Exam - Practice Problems

Problem 1. You need to manufacture a cylindrical pot, without a top, with a volume of 1 ft^3. The cylindrical part of the pot is to be made of aluminum, the bottom of copper. Copper is five times as expensive as aluminum. What dimensions would minimize the total cost of the pot?

Problem 2. If y is defined implicitly as a function of x through the equation

$$2x^2 - 3xy + 5y^2 = 25,$$

calculate d^2y/dx^2.

Problem 3. Show that the curve with equation

$$x^{1/3} + y^{1/3} = 1$$

is never concave down in the first quadrant of the xy-plane.

Problem 4. Compute

$$\lim_{x\to\infty} x\left[\left(1+\frac{1}{x}\right)^x - e\right].$$

[*Hint:* Use the definition given in class of the limit at infinity, i.e. make the substitution $x = 1/u$ and take the limit as $u \to 0^+$.]

Problem 5. Does the limit

$$\lim_{x\to 0}(e^{1/x} - 1)\tan x$$

exist? Justify your answer.

Problem 6. Evaluate

$$\lim_{x\to\infty}\left(\frac{x^2}{x+2} - \frac{x^3}{x^2+3}\right).$$

Problem 7. You are at the southernmost point of a circular lake of radius 1 mi. Your plan is to swim a straight course to another point on the point on the shore of the lake, then jog to the northernmost point. You can jog twice as fast as you can swim. What route gives the minimum time required for your journey?

Problem 8. Calculate the first three derivatives of

$$g(t) = \frac{8}{(3-t)^{3/2}}.$$

Conjecture a general formula for the nth derivative.

Problem 9. Sketch the graph of the function

$$f(x) = \frac{x}{x^2 - x - 2}$$

indicating all critical points, points of inflection, and vertical and horizontal asymptotes.

Problem 10. Sketch the graph of the function

$$g(x) = \begin{cases} x \ln |x| & , x \neq 0 \\ 0 & , x = 0. \end{cases}$$

Problem 11. Show that the function

$$h(x) = \frac{1}{x^2 + 2x + 2}$$

is defined for all real x and find, with proof, its absolute maximum value.

Problem 12. Suppose that it costs $1 + (0.0003)v^{3/2}$ dollars per mile to operate a truck at v miles per hour. If there are additional costs (such as the driver's pay) of 10 dollars per hour, what speed would minimize the total cost of a 1000 mile trip?

Problem 13. Consider the function

$$r(x) = |\ln x|^{|\ln x|},$$

which is defined for all $x > 0$, $x \neq 1$.

(a) Show that the discontinuity in $r(x)$ at $x = 1$ is removable. How must we define $r(1)$ in order to make $r(x)$ continuous for all $x > 0$?

(b) If we define $r(1)$ as in part (a) does $r'(1)$ exist?

Problem 14. Sketch the graph of the function

$$q(x) = (x-1)e^{-x^2}.$$