Math 1311 Fall 2006

Calculus I

Final Exam - Practice Problems

Problem 1. Use Riemann sums to compute

$$\int_0^2 2 - \frac{2x}{3}\, dx.$$

Use the Fundamental Theorem of Calculus to check your answer.

Problem 2. Let $f(x)$ be the function whose graph is shown below.

$$g(x) = \int_1^x f(t)\, dt$$

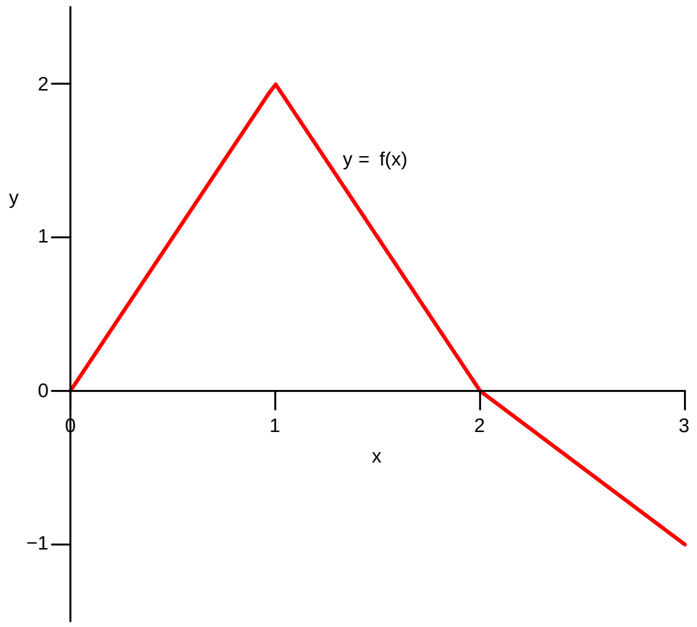
where the function $f(x)$ has the graph shown below.

(a) If

$$g(x) = \int_1^x f(t)\, dt,$$

compute $g(0), g(1), g(2)$ and $g(3)$.

(b) Compute the average value of $f(x)$ on the interval $[0, 3]$.



Problem 3. Make the substitution $u = r - 3$ in the definite integral

$$\int_2^4 r\sqrt{1-(r-3)^2}\,dr$$

then evaluate the resulting definite integral using geometry.

Problem 4. Evaluate the following indefinite integrals.

(a)

$$\int (x^2 + 2x + 1)^9 (x+1)\,dx$$

(b)

$$\int \tan y\,dy$$

(c)

$$\int \sqrt{x}(1+\sqrt{x})^{20}\,dx$$

Problem 5. The driver of a car traveling in a straight line slams on its brakes at time $t = 0$. If the car experiences a constant deceleration of 40 ft/s^2 and skids 180 ft before coming to a stop, how fast was the car traveling when the brakes were first applied?

Problem 6. Find the derivative of the function

$$G(x) = \int_{-x^2}^{x} \sqrt{t^2 - 2t + 2}\,dt.$$

Problem 7. Use the fundamental theorem of calculus to find a continuous function $g(x)$ which satisfies the equation

$$x + \sin x = \int_0^x (g(t))^2\,dt.$$

Problem 8. Evaluate

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 - \left(\frac{i}{n}\right)^5\right) \frac{1}{n}$$

by interpreting the limit as a definite integral.

Problem 9. Evaluate the following definite integrals.

(a)

$$\int_0^1 \frac{dt}{3-2t}$$

(b)

$$\int_0^2 (1-x^2)^2\,dx$$

(c)

$$\int_{-1}^1 e^{3x}(1-e^{3x})^4\,dx$$

(d)

$$\int_0^{\pi/4} \sec^2\theta\,d\theta$$

Problem 10.

(a) Show that

$$\frac{d}{dx}(x\ln x - x) = \ln x.$$

(b) Use the result of part (a) to compute

$$\int_1^{e^5} \ln x\,dx.$$

Problem 11. Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi$.

Problem 12. Evaluate

$$\sum_{k=1}^{100}\left(\frac{1}{k} - \frac{1}{k+1}\right).$$

[*Hint:* Write out the first few terms of the sum.]

Problem 13. Find the area trapped between the curves $y = x^{3/2}$ and $y = 2 - x^2$.

Problem 14. Set up, but *do not evaluate*, the integral giving the arc length for one arch of the curve $y = \sin x$.

Problem 15. Carefully state both parts of the Fundamental Theorem of Calculus. Use the second part of the theorem to prove the first.

Problem 16. Use definite integrals to find a function $f(x)$ satisfying

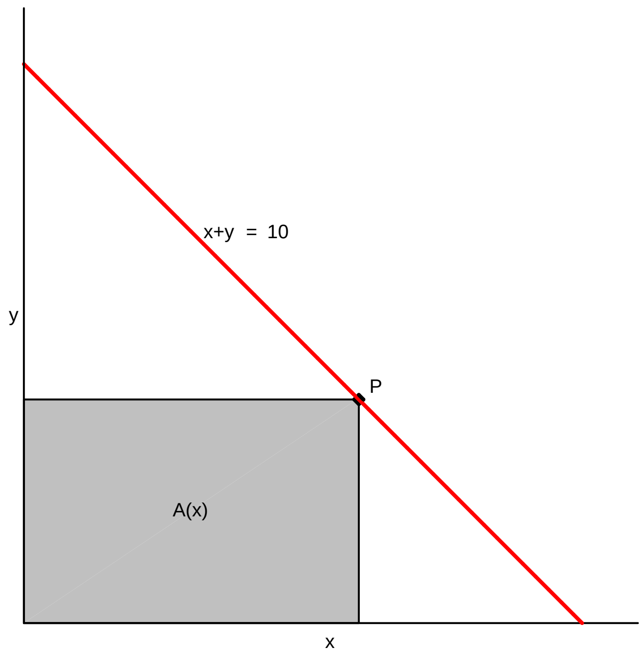
$$f'(x) = \frac{1}{\ln x}\ ,\ f(2) = 0.$$

Problem 17. Evaluate

$$\lim_{x \to 0} \frac{\int_0^x \frac{\sin t}{t}\, dt}{x}.$$

[*Hint:* Use L'Hospital's rule and the Fundamental Theorem of Calculus.]

Problem 18. Consider a rectangle inscribed in the first-quadrant region that lies between the x-axis and the line $y = 10 - x$, as shown in the figure below. Express this area as a function $A(x)$ of the x-coordinate of its vertex P on the line. Find the maximum and average values of $A(x)$.



Problem 19. Let

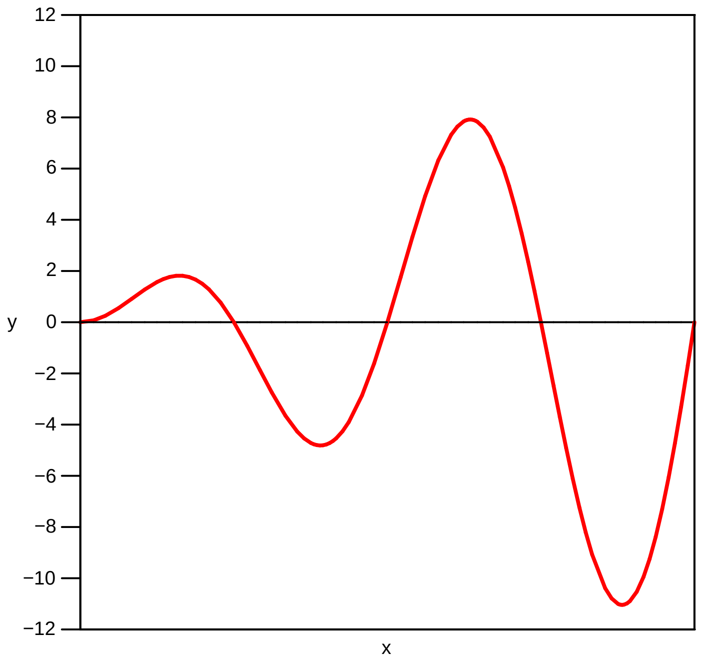
$$f(x) = \int_2^x \frac{t}{t^2 + 1} dt.$$

(a) Compute $f'(x)$ by finding a closed form expression for $f(x)$ and differentiating.

(b) Verify your computation in part (a) by differentiating $f(x)$ directly using the Fundamental Theorem of Calculus.

Problem 20. The figure below shows the graph of the function $f(x) = x \sin x$ on the interval $[0, 4\pi]$. Let

$$g(x) = \int_0^x f(t)\, dt.$$



(a) Find the values of x at which $g(x)$ has local maximum and minimum values on $[0, 4\pi]$.

(b) Where does $g(x)$ attain its global maximum and minimum values on $[0, 4\pi]$?

(c) Which points on the graph $y = f(x)$ correspond to inflection points on the graph of $y = g(x)$?

(d) Sketch a rough graph of $y = g(x)$.