Problem 1. Use Riemann sums to compute
\[ \int_0^2 2 - \frac{2x}{3} \, dx. \]
Use the Fundamental Theorem of Calculus to check your answer.

Problem 2. Let \( f(x) \) be the function whose graph is shown below.
\[ g(x) = \int_1^x f(t) \, dt \]
where the function \( f(x) \) has the graph shown below.
(a) If
\[ g(x) = \int_1^x f(t) \, dt, \]
compute \( g(0), g(1), g(2) \) and \( g(3) \).
(b) Compute the average value of \( f(x) \) on the interval \([0, 3]\).
Problem 3. Make the substitution $u = r - 3$ in the definite integral
$$\int_2^4 (r - 3)^2 dr$$
then evaluate the resulting definite integral using geometry.

Problem 4. Evaluate the following indefinite integrals.
(a) \[\int (x^2 + 2x + 1)^{9}(x + 1) \, dx\]
(b) \[\int \tan y \, dy\]
(c) \[\int \sqrt{x}(1 + \sqrt{x})^{20} \, dx\]

Problem 5. The driver of a car traveling in a straight line slams on its brakes at time $t = 0$. If the car experiences a constant deceleration of 40 ft/s$^2$ and skids 180 ft before coming to a stop, how fast was the car traveling when the brakes were first applied?

Problem 6. Find the derivative of the function
$$G(x) = \int_{-x^2}^{x} \sqrt{t^2 - 2t + 2} \, dt.$$ 

Problem 7. Use the fundamental theorem of calculus to find a continuous function $g(x)$ which satisfies the equation
$$x + \sin x = \int_{0}^{x} (g(t))^2 \, dt.$$ 

Problem 8. Evaluate
$$\lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 - \left( \frac{i}{n} \right)^5 \right) \frac{1}{n}$$
by interpreting the limit as a definite integral.

Problem 9. Evaluate the following definite integrals.
(a) \[\int_{0}^{1} \frac{dt}{3 - 2t}\]
Problem 10. 
(a) Show that 
\[ \frac{d}{dx}(x \ln x - x) = \ln x. \]
(b) Use the result of part (a) to compute 
\[ \int_{e^5}^1 \ln x \, dx. \]

Problem 11. Find the area of the region bounded by the curves \( y = \sin x, \) \( y = \cos x, \) \( x = 0 \) and \( x = \pi. \)

Problem 12. Evaluate 
\[ \sum_{k=1}^{100} \left( \frac{1}{k} - \frac{1}{k+1} \right). \]
[Hint: Write out the first few terms of the sum.]

Problem 13. Find the area trapped between the curves \( y = x^{3/2}, \) and \( y = 2 - x^2. \)

Problem 14. Set up, but do not evaluate, the integral giving the arc length for one arch of the curve \( y = \sin x. \)

Problem 15. Carefully state both parts of the Fundamental Theorem of Calculus. Use the second part of the theorem to prove the first.

Problem 16. Use definite integrals to find a function \( f(x) \) satisfying 
\[ f'(x) = \frac{1}{\ln x}, \quad f(2) = 0. \]
Problem 17. Evaluate
\[
\lim_{x \to 0} \int_0^x \frac{\sin t}{t} \, dt.
\]
[Hint: Use L’Hospital’s rule and the Fundamental Theorem of Calculus.]

Problem 18. Consider a rectangle inscribed in the first-quadrant region that lies between the x-axis and the line \( y = 10 - x \), as shown in the figure below. Express this area as a function \( A(x) \) of the x-coordinate of its vertex \( P \) on the line. Find the maximum and average values of \( A(x) \).

\[ \begin{align*}
x+y &= 10 \\
y = A(x) \\
P \\
x \\
\end{align*} \]

Problem 19. Let
\[
f(x) = \int_2^x \frac{t}{t^2 + 1} \, dt.
\]
(a) Compute \( f'(x) \) by finding a closed form expression for \( f(x) \) and differentiating.

(b) Verify your computation in part (a) by differentiating \( f(x) \) directly using the Fundamental Theorem of Calculus.
Problem 20. The figure below shows the graph of the function $f(x) = x \sin x$ on the interval $[0, 4\pi]$. Let

$$g(x) = \int_0^x f(t) \, dt.$$

(a) Find the values of $x$ at which $g(x)$ has local maximum and minimum values on $[0, 4\pi]$.

(b) Where does $g(x)$ attain its global maximum and minimum values on $[0, 4\pi]$?

(c) Which points on the graph $y = f(x)$ correspond to inflection points on the graph of $y = g(x)$?

(d) Sketch a rough graph of $y = g(x)$. 