Math 1311
Calculus I
Slope Problems

In trying to define the "slope" of the graph of $y = f(x)$ at $x = a$ we introduced the difference quotient

$$\frac{f(a+h) - f(a)}{h} \tag{1}$$

which gives the slope of the secant line passing through $(a, f(a))$ and $(a+h, f(a+h))$. As h approaches 0, these points move ever closer to one another, and we argued intuitively that we expect the slopes to approach some limiting value. In fact, we defined the "slope" of the graph of $y = f(x)$ at $x = a$ to be the limit of the slopes of secant lines to the graph:

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Below we will experimentally verify that, at least for some functions, this definition makes sense.

In the following two exercises, use a calculator or computer to investigate numerically the "slope" m_a of the graph of $y = f(x)$ by calculating the difference quotient (1) for $h = 0.1, 0.01, 0.001, \ldots$. Check the resulting value of m_a by plotting both the graph of $y = f(x)$ and the graph of the *tangent line* at $(a, f(a))$, i.e./ line through $(a, f(a))$ with slope m_a.

Exercise 1. $f(x) = \frac{1}{x}, a = -\frac{1}{2}$

Exercise 2. $f(x) = \sqrt{25 - x^2}, a = 3$

Exercise 3. Try the same thing for the function $f(x) = \sqrt{x}$ with $a = 0$. What happens? Can you explain this behavior?