Calculus I — Exam 2 Practice Problems

Fall 2007

1. If two resistors with resistances R_1 and R_2 are wired in parallel, the resulting net resistance R is determined by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Estimate the change in R if R_1 is increased from 2 to 2.1 and R_2 is increased from 4 to 4.2.

2. Show that for small x we have

$$\sin x \approx x.$$

3. Suppose that f is a differentiable function with $f(2) = 5$ and $f'(x) \leq 3$ for all x. Show that $f(x) \leq 3x - 1$ for all $x \geq 2$. [*Hint:* Use the Mean Value Theorem.]

4. Show that if $f'(x) \neq 1$ for all real x, then the equation $f(x) = x$ has at most one solution.

5. Find the intervals of increase or decrease for the following functions.

(a)

$$f(x) = x + \cos x, \ -2\pi \leq x \leq 2\pi$$

(b)

$$g(x) = x\sqrt{x+3}$$

(c)

$$h(x) = (x^2 - 1)^3$$

6. Suppose that it costs $1 + (0.0003)v^{3/2}$ dollars per mile to operate a truck at v miles per hour. If there are additional costs (such as the driver's pay) of 10 dollars per hour, what speed would minimize the total cost of a 1000 mile trip?

7. Find an equation of the line through the point $(2, 3)$ that cuts off the least area from the first quadrant.

8. If

$$f(x) = \frac{x^2 + 4x + 4}{3x^2 + 12x + 21}$$

then

$$f'(x) = \frac{2x + 4}{(x^2 + 4x + 7)^2}$$

and

$$f''(x) = \frac{-6x^2 - 24x - 18}{(x^2 + 4x + 7)^3}.$$

Use this information to carefully sketch the graph of $y = f(x)$. Be sure to identify all roots, critical points, intervals of increase and decrease, local maxima and minima, intervals of concavity, inflection points, and vertical and horizontal asymptotes.

9. If

$$h(x) = x^{-2}e^{x}$$

carefully sketch the graph of $y = h(x)$. Be sure to identify all roots, critical points, intervals of increase and decrease, local maxima and minima, intervals of concavity, inflection points, and vertical and horizontal asymptotes.

10. Evaluate the following limits.

(a)

$$\lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$$

(b)

$$\lim_{t \to 0} \frac{e^{t/6} - e^{t/3}}{t}$$

(c)

$$\lim_{x \to 0} (1 - 2x)^{1/x}$$