Calculus I Fall 2007

1. If $\Delta x = 2/n$ and $x_i = 1 + i\Delta x$, evaluate

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{1}{x_i^2} - x_i + 6 \right) \Delta x$$

by expressing the limit as a definite integral and applying the Fundamental Theorem of Calculus.

2. Use Riemann sums to evaluate the definite integral

$$\int_{-1}^{1} (x^2 - 2x) \, dx.$$

3. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.

(a)
$$\int_{e}^{e^{2}} \frac{(\ln x)^{2}}{x} dx$$

(b) $\int_{0}^{3} (1+3y-y^{2}) dy$
(c) $\int_{1}^{9} \frac{\sqrt{u}-2u^{2}}{u} du$
(d) $\int_{\pi/4}^{\pi/3} (1+\cot\theta)^{3} \csc^{2}\theta \, d\theta$

(e)
$$\int_{-3}^{0} \frac{x}{\sqrt{1-x}} dx$$
 [*Hint:* Try the substitution $u = 1 - x$.]

4. A model rocket is launched vertically from the ground at time t = 0. It is initially at rest, and the engine in the rocket burns for 5 s, providing the rocket with a constant upward acceleration of 8 ft/s².

- (a) At what height is the rocket when its engine burns out? What is its velocity at that point?
- (b) The rocket continues upward after the engine burns out, subject only the downward pull of gravity. What is the maximum height that the rocket reaches? Remember, the acceleration due to gravity is 32 ft/s².
- 5. For the function f(x), whose graph is shown below, define

$$g(x) = \int_0^x f(t) \, dt$$

for x in [0, 7].



Figure 1: Graph of y = f(x).

- (a) Compute g(0), g(2), g(5) and g(7).
- (b) Find the critical points of g(x) and determine the intervals on which g(x) is increasing and decreasing. Use this to classify the critical points.
- (c) Find the absolute maximum and minimum values of g(x) on the interval [0,7].
- (d) Find the intervals on which g(x) is concave up and concave down.
- (e) Carefully sketch the graph of g(x).
- **6.** Find dy/dx if

$$y = g(x) = \int_{x}^{2x} \frac{u^2 - 1}{u^2 + 1} du.$$

- 7. Find the average value of the function f(x) = |x| on the interval [-1, 2].
- 8. Find the area of the region enclosed by the curves $y = 2x x^2$ and $y = x^2 4x$.