

# MATH 1311 FALL 2006

## CALCULUS I

### FINAL EXAM

SATURDAY, DECEMBER 9, 6:30 PM - 9:30 PM

YOUR NAME (PLEASE PRINT):

**Instructions:** This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

**The Honor Code requires that you neither give nor receive any aid on this exam.**

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by circling the word “pledged” below and signing your name in the space provided:

Pledged: \_\_\_\_\_

Problem	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
Points	10	15	20	15	25	10	20
Score							

Problem	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>Total</b>
Points	20	15	10	10	15	15	200
Score							

**Problem 1.** If  $\Delta x = 2/n$  and  $x_i = 1 + i\Delta x$ , express

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (3x_i^2 - x_i + 6) \Delta x$$

as a definite integral. Evaluate this integral.

**Problem 2.** Use Riemann sums to evaluate the definite integral

$$\int_{-1}^1 (x - 2) dx.$$

**Problem 3.** Evaluate the following definite integrals.

(a) 
$$\int_e^{e^2} \frac{(\ln x)^2}{x} dx$$

(b) 
$$\int_0^3 (1+3y-y^2) dy$$

(c) 
$$\int_1^9 \frac{\sqrt{u} - 2u^2}{u} du$$

(d)  $\int_{\pi/4}^{\pi/3} (1 + \cot \theta)^3 \csc^2 \theta \, d\theta$

(e)  $\int_{-3}^0 \frac{x}{\sqrt{1-x}} \, dx$  [*Hint:* Try the substitution  $u = 1 - x$ .]

**Problem 4.** A model rocket is launched vertically from the ground at time  $t = 0$ . It is initially at rest, and the engine in the rocket burns for 5 s, providing the rocket with a constant upward acceleration of  $8 \text{ ft/s}^2$ .

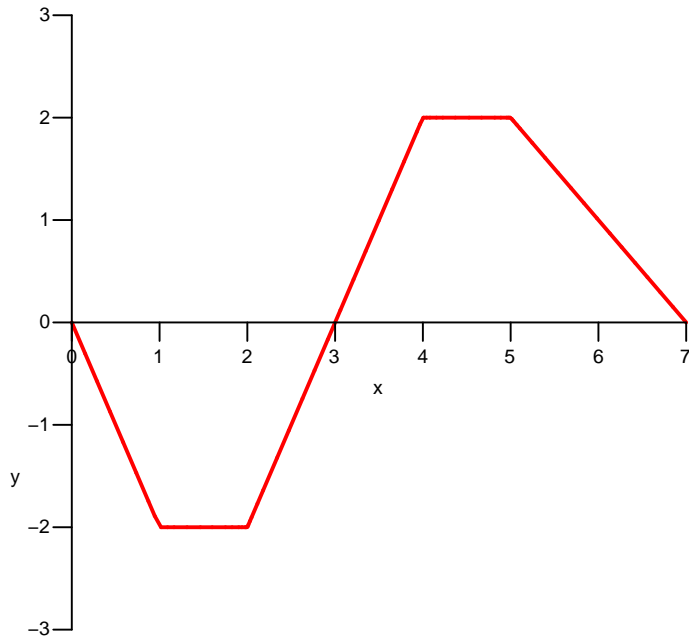
(a) At what height is the rocket when its engine burns out? What is its velocity at that point?

(b) The rocket continues upward after the engine burns out, subject only the downward pull of gravity. What is the maximum height that the rocket reaches? Remember, the acceleration due to gravity is  $32 \text{ ft/s}^2$ .

**Problem 5.** For the function  $f(x)$ , whose graph is shown below, define

$$g(x) = \int_0^x f(t) dt$$

for  $x$  in  $[0, 7]$ .



(a) Compute  $g(0)$ ,  $g(2)$ ,  $g(5)$  and  $g(7)$ .

(b) Find the critical points of  $g(x)$  and determine the intervals on which  $g(x)$  is increasing and decreasing. Use this to classify the critical points.

(c) Find the absolute maximum and minimum values of  $g(x)$  on the interval  $[0, 7]$ .



(d) Find the intervals on which  $g(x)$  is concave up and concave down.

(e) Carefully sketch the graph of  $g(x)$ .

**Problem 6.** Find the area of the region enclosed by the curves  $y = 2x - x^2$  and  $y = x^2 - 4x$ .

**Problem 7.** Evaluate the following limits.

(a)  $\lim_{x \rightarrow 2^+} \frac{1 - x}{x^2 - 5x + 6}$

(b)  $\lim_{t \rightarrow 9} \frac{9 - t}{3 - \sqrt{t}}$

(c)  $\lim_{t \rightarrow \infty} \frac{5t^{16} + 6t^9 - 23t^2 + 8}{9t^{16} - t^{15} + t^{13}}$

$$(d) \lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

$$(e) \lim_{x \rightarrow 3} \frac{\int_3^x \sqrt{t^2 + 1} dt}{x - 3}$$

**Problem 8.** Find  $dy/dx$  if:

(a)  $y = g(x) = \int_x^{2x} \frac{u^2 - 1}{u^2 + 1} du.$

(b)  $x^3 + 3xy + y^3 = 1.$

(c)  $y = e^{x \sin x}.$

$$(d) \quad y = \sqrt{\frac{x+1}{x-1}}.$$

$$(e) \quad y = x^5 - 7x^2 + x^{-1/2} + x^{-2} - 5 \ln x.$$

**Problem 9.** At noon, ship  $A$  is 90 km west of ship  $B$ . Ship  $A$  is sailing east at 30 km/h and ship  $B$  is sailing north at 20 km/h. How fast is the distance between the ships changing at 2 PM?

**Problem 10.** A fish is swimming at a speed  $v$  relative to the water, against a current with speed 10 ft/s (so  $v > 10$ ). To swim a distance  $L$  against the current, the fish to must expend an amount of energy proportional to

$$E(v) = v^3 \frac{L}{v - 10}.$$

Find the speed  $v$  which minimizes the energy used by the fish. Be sure to carefully justify your answer!



**Problem 11.** Find the absolute maximum and minimum values achieved by the function

$$h(x) = x^{2/3}(x - 4)^2$$

on the interval  $[-1, 2]$ .

**Problem 12.**

(a) If  $f(x)$  is a function defined in a neighborhood of  $x = a$ , write down the limit definition of  $f'(a)$ .

(b) If  $f(x) = x^2 - x$ , use the limit definition of the derivative to compute  $f'(x)$ . You *may not* use l'Hospital's rule to evaluate the limit.

**Problem 13.** Use the Intermediate Value Property to show that the equation

$$\frac{1}{4}x^4 = \frac{7}{3}x^3 - 6x^2 + 5$$

has at least one solution in the interval  $(1, 2)$ . Then use the Mean Value Theorem to show that there is only one such solution.

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