Calculus I
Fall 2009

Exam 1
Practice Problems

Exercise 1. Find a formula for a function f whose graph passes through $(0,0)$, has vertical asymptotes $x=-1$ and $x=2$ and has horizontal asymptote $y=-3$.

Exercise 2. Find the limit.

a. $\displaystyle\lim_{t\to -3}\frac{t^2-9}{t^2+2t-3}$

b. $\displaystyle\lim_{x\to 4}\frac{x-\sqrt{2x+8}}{x^3-4x^2}$

c. $\displaystyle\lim_{x\to\infty}\sqrt{x^2+x+1}-\sqrt{x^2-x}$

d. $\displaystyle\lim_{w\to-\infty}\frac{\sqrt{w^2-9}}{2w-6}$

e. $\displaystyle\lim_{z\to 1}\left(\frac{1}{z-1}+\frac{1}{z^2-3z+2}\right)$

f. $\displaystyle\lim_{\theta\to 0}\frac{\theta^2}{\cos\theta-1}$

g. $\displaystyle\lim_{x\to 0^+}e^{1/x}$

h. $\displaystyle\lim_{x\to 0^-}e^{1/x}$

Exercise 3. Show that the polynomial $p(x)=x^3-4x+1$ has three real roots. [*Hint:* Show that $p(x)$ has at least 3 sign changes and use the Intermediate Value Theorem]

Exercise 4. Differentiate $f(x)=\dfrac{1}{\sqrt{x+2}}$ *using the definition of the derivative.*

Exercise 5. Find the parabola with equation $y=ax^2+bx$ whose tangent line at $(1,1)$ has the equation $-3x+y=-2$.

Exercise 6. Find constants a and b so that the function

$$r(x)=\begin{cases}-3x+2 & \text{if } x\le -1,\\ x^2+ax+b & \text{if } x>-1\end{cases}$$

is differentiable everywhere.

Exercise 7. Evaluate the limit

$$\lim_{h \to 0} \frac{(1+h)^{1000} - 1}{h}.$$

Exercise 8. Find $\dfrac{dy}{dx}$.

a. y $= \sqrt{x} \cos \sqrt{x}$

b. $y = \ln\left(x^3 - 7x + 3\right)$

c. $y = \dfrac{e^x}{x^2+1}$

d. $y = |x|^3$

e. $\tan\left(e^{-x^2}\right)$

f. $y = e^{\alpha x} \sin \beta x$

g. $xy^4 + x^2 y = x + 3y$

h. $y = \dfrac{\sqrt{x+1}(2-x)^5}{(x+3)^7}$

Exercise 9. Show that the triangle formed by the coordinates axes and the tangent line to the curve $xy = 1$ at the point $(a, 1/a)$ always has an area equal to 2.

Exercise 10. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$ and $f'(5) = 11$. Find $h'(2)$ and $F'(2)$.