Calculus I
Fall 2009

Exam 3
Practice Problems

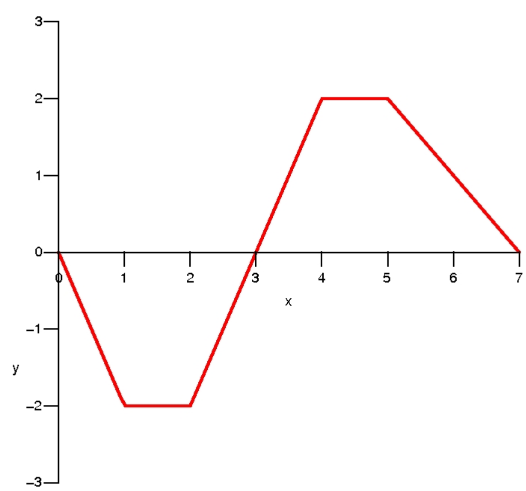
Exercise 1. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

Exercise 2. A box with a square base and open top is to be made using exactly 1200 cm^2 of material. What is the largest possible volume of such a box?

Exercise 3. A driver involved in an accident claims he was going only 25 mi/h. When investigators tested his car, they found that the maximum deceleration its brakes could provide was 15 ft/s^2. Given that the skid marks left by the car at the scene of the accident were 210 ft long, is the driver telling the truth?

Exercise 4. For the function $f(x)$, whose graph is shown below, define

$$g(x) = \int_0^x f(t)\,dt.$$



a. At what values of x do the local maximum and minimum values of g occur?

b. Find the absolute maximum and minimum values of g on the interval $[0, 7]$.

c. Find the intervals on which g is concave up and concave down.

d. Carefully sketch the graph of g.

Exercise 5. Evaluate the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i\sqrt{1+i^2/n^2}}{n^2}.$$

[*Hint:* Express the limit as a definite integral.]

Exercise 6. If f is a continuous function such that

$$\int_0^x f(t)\,dt = xe^{2x} + \int_0^x e^{-t} f(t)\,dt$$

for all x, find an explicit formula for $f(x)$. [*Hint:* Apply the Fundamental Theorem of Calculus to get rid of the integrals.]

Exercise 7. Evaluate the integral.

a. $\displaystyle\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du$

b. $\displaystyle\int_0^1 \sin(3\pi t)\,dt$

c. $\displaystyle\int \frac{x^2}{\sqrt{x-3}}\,dx$

d. $\displaystyle\int_1^{10} \frac{x}{x^2-4}\,dx$

e. $\displaystyle\int \frac{\cos(\ln x)-\sin(\ln x)}{x}\,dx$

f. $\displaystyle\int_0^{\pi/4} (1+\tan\theta)^3 \sec^2\theta\,d\theta$

g. $\displaystyle\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

h. $\displaystyle\int_0^4 \left|\sqrt{x}-1\right|\,dx$

Exercise 8. Find the area of the region bounded by the given curves.

a. $y = \sin(\pi x/2)$, $y = x^2 - 2x$

b. $y = \sqrt[n]{x}$, $y = 2x - 1$, $y = 0$ $(n > 1)$

c. $y = x^2 - 4$, $y = x^3 - 4x$

Exercise 9. Find the volume of the solid whose base is the region enclosed by the parabola $y = 1-x^2$ and the x-axis, and whose cross-sections perpendicular to the y-axis are semicircles.

Exercise 10. Find the volume of the "cap" of a sphere with radius r and height h (see the diagram below).