

 $\begin{array}{c} {\rm Calculus} \ {\rm I} \\ {\rm Fall} \ 2009 \end{array}$

FINAL EXAM PRACTICE PROBLEMS 1

Exercise 1. Evaluate the following limits, if they exist.

a. $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$ **b.** $\lim_{x \to 0} e^{1/x}$ **c.** $\lim_{x \to 3} (2x + |x - 3|)$ **d.** $\lim_{x \to -\infty} \sqrt{x^2 + x + 1} + x$ **e.** $\lim_{t \to 2} \frac{t^2 - 4}{t^3 - 8}$ **f.** $\lim_{\theta \to 0} \frac{\sin \theta}{\theta + \tan \theta}$

Exercise 2. Find $\frac{dy}{dx}$.

a.
$$y = e^{\pi}$$

b. $y = \ln \left| \frac{x^2 - 4}{2x + 5} \right|$
c. $y = e^{-x}(x^2 - 2x + x)$
d. $y = \sqrt{\sin \sqrt{x}}$
e. $y = e^{x \tan x}$
f. $xy^4 + x^2y = x + 3y$

Exercise 3. A 10 foot ladder rests against a vertical wall. The base of the ladder begins to slide away from the wall at a rate of 2 ft/s.

- **a.** How fast is the top of the ladder falling when the base of the ladder is 6 ft from the wall?
- **b.** As the top of the ladder approaches the ground, what does its speed approach? Is this realistic? Why or why not?

Exercise 4. Find the local and absolute extreme values of $f(x) = (x^2 + 2x)^3$ on the interval [-2, 1].

Exercise 5. Carefully sketch the graph of $y = e^{2x-x^2}$.

Exercise 6. A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal?