Calculus I
Spring 2008

Exam 1 Practice Problems

1. The function

$$q(z) = \frac{\tan z}{\sin 2z}$$

is not defined, and therefore not continuous, at $z = 0$. Is the discontinuity removable?

2. If $f(x) = x^3 - x^2 + x$, use the Intermediate Value Property to show that there is a number c so that $f(c) = 10$. Be sure to explain why you may apply the Intermediate Value Property.

3. Let

$$r(x) = \frac{x^2 + 4x + 3}{x^4 - x^3 - 2x^2}.$$

a. Identify the points at which $r(x)$ is discontinuous.

b. Describe the behavior of $r(x)$ near each of its discontinuities. That is, determine whether or not they are removable and, if not, say as much as you can about the one sided limits at those points.

4. Evaluate the following limits, if they exist. If a limit does not exist, determine whether or not it is infinite.

(a) $\displaystyle\lim_{z \to 1} \frac{z^2 + z + 1}{\sqrt{z+1} - 2}$

(b) $\displaystyle\lim_{x \to a} 7x^3 - 5x + \frac{\cos x}{x^2 + 1}$

(c) $\displaystyle\lim_{x \to \pi^+} \frac{x}{\sin x}$

(d) $\displaystyle\lim_{w \to 2^-} \frac{w^2 + w - 6}{w^2 - w - 2}$

(e) $\displaystyle\lim_{x \to 3} \frac{x + 4}{x^2 + x - 12}$

(f) $\displaystyle\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$

5. Suppose the graph of the function $f(x)$ has tangent line $y = 4x - 1$ at the point $(1,3)$ and that the graph of the function $g(x)$ has tangent line $y = 6x - 4$ at the point $(1,2)$. Use this information to find an equation for the tangent line to the graph of $f(x)g(x)$ when $x = 1$. Do the same for $f(x)/g(x)$.

6. Use the *limit definition of the derivative* to compute $g'(2)$ if $g(x) = \dfrac{1}{x^2+3}$.

7. Let

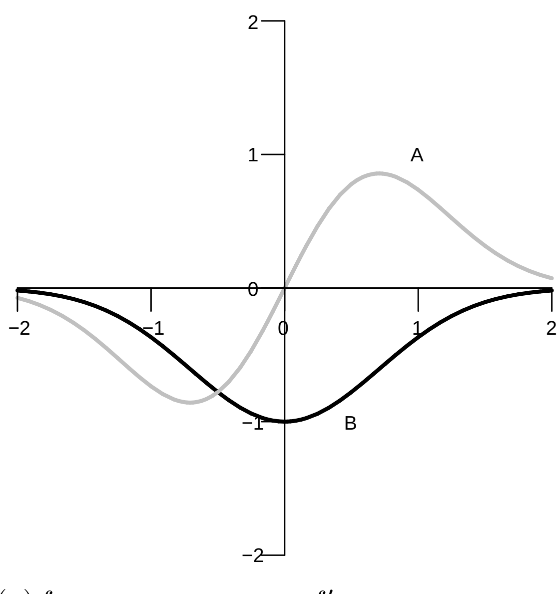
$$f(x) = \frac{x^3}{1+x^2}$$

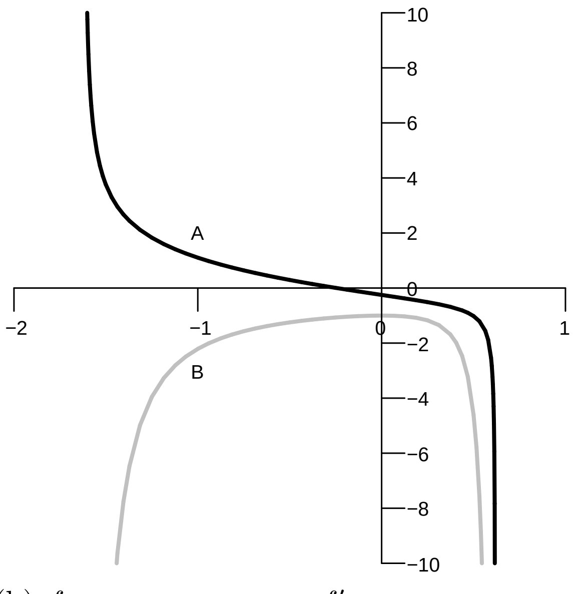
Find the point(s) on the graph of $y = f(x)$ at which the tangent line is parallel to the line with equation $3x + 3y = 1$.

8. Find the absolute maximum and minimum values attained by the function $g(x) = 2x^3 + 3x^2 - 12x + 2$ on the interval $[-3, 0]$.

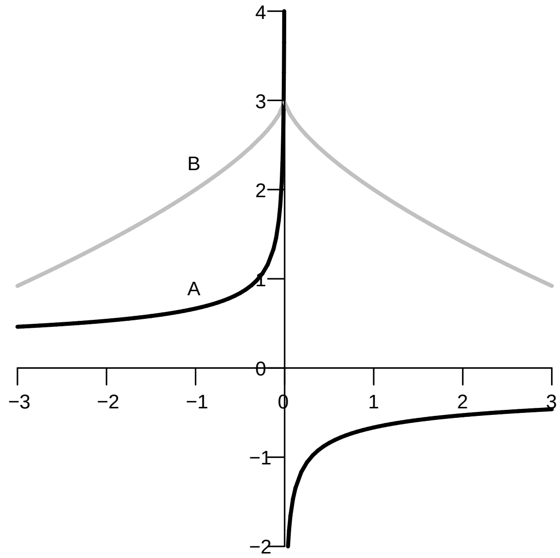
9. A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2,3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

10. In each figure below the graph of a function $f(x)$ is shown together with the graph of its derivative $f'(x)$. In each case, identify which curve is the graph of $f(x)$ and which is the graph of $f'(x)$.

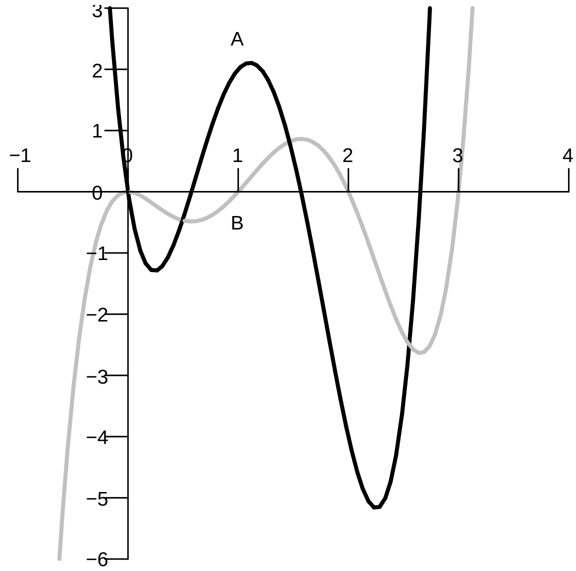




(a)$f =$ ____________, $f' =$ ____________ (b) $f =$ ____________, $f' =$ ____________



(c) $f =$ ____________, $f' =$ ____________



(d) $f =$ ____________, $f' =$ ____________