

1. Sketch the graphs of the following functions, indicating all critical points, points of inflection, and vertical and horizontal asymptotes.

(a) $f(x) = \frac{x^2 + 4x + 4}{3x^2 + 12x + 21}$

(b) $g(x) = (x-1)e^{-x^2}$.

(c) $h(x) = \frac{x}{x^2 - x - 2}$.

(d) $r(x) = \frac{\ln x + 1}{x}$.

2. Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$

(b) $\lim_{t \rightarrow 0} \frac{e^{t/6} - e^{t/3}}{t}$

(c) $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$

(d) $\lim_{x \rightarrow \infty} \left(\frac{x^2}{x+2} - \frac{x^3}{x^2+3} \right)$.

(e) $\lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}}$

(f) $\lim_{x \rightarrow \infty} \frac{1+x^3+x^5}{x-x^2+x^4}$

(g) $\lim_{x \rightarrow \infty} \sqrt{4x^2+3x-5} - 2x$

(h) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{\sqrt{x+9} - 3}$

3. Compute

$$\lim_{x \rightarrow \infty} x \left[\left(1 + \frac{1}{x} \right)^x - e \right].$$

[Hint: Use the definition given in class of the limit at infinity, i.e. make the substitution $x = 1/u$ and take the limit as $u \rightarrow 0^+$.]