Calculus I — Final Exam Practice Problems

Spring 2008

1. Evaluate the following integrals.

(a) $\displaystyle\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du$

(b) $\displaystyle\int_{\pi/4}^{\pi/3} (1+\cot\theta)^3 \csc^2\theta\,d\theta$

(c) $\displaystyle\int_{-3}^{0} \frac{x}{\sqrt{1-x}}\,dx$

(d) $\displaystyle\int_{-\pi/4}^{0} \tan\theta\,d\theta$

(e) $\displaystyle\int_0^1 \frac{dt}{3-2t}$

(f) $\displaystyle\int_{-1}^{1} e^{3x}(1-e^{3x})^4\,dx$

(g) $\displaystyle\int (x^2+2x+1)^9(x+1)\,dx$

(h) $\displaystyle\int \sqrt{x}(1+\sqrt{x})^{20}\,dx$

(i) $\displaystyle\int_{-3}^{2} |t|\,dt$

(j) $\displaystyle\int_e^{e^e} \frac{1}{x\ln x}\,dx$

(k) $\displaystyle\int_0^1 y^3(1+y)^2\,dy$

(l) $\displaystyle\int_1^{8/7} \frac{1}{x^2}\sqrt[3]{1-\frac{1}{x}}\,dx$

2. Make the substitution $u = r - 3$ in the definite integral

$$\int_2^4 r\sqrt{1-(r-3)^2}\,dr$$

then evaluate the resulting definite integral using geometry.

3. If $\Delta x = 2/n$ and $x_i = 1 + i\Delta x$, evaluate

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{1}{x_i^2} - x_i + 6\right)\Delta x.$$

4. Use Riemann sums to evaluate the definite integral

$$\int_{-1}^{1}(x^2-2x)\,dx.$$

5. Find the derivative of the function

$$G(x) = \int_{-x^2}^{x} \sqrt{t^2 - 2t + 2}\, dt.$$

6. Use the fundamental theorem of calculus to find a continuous function $g(x)$ which satisfies the equation

$$x + \sin x = \int_0^x (g(t))^2\, dt.$$

7. Evaluate

$$\lim_{x \to 0} \frac{1}{x} \int_0^x \sqrt{\cos t}\, dt.$$

[*Hint:* Rewrite the product as a quotient and use the Fundamental Theorem of Calculus.]

8. A model rocket is launched vertically from the ground at time $t = 0$. It is initially at rest, and the engine in the rocket burns for 5 s, providing the rocket with a constant upward acceleration of 8 ft/s^2.

(a) At what height is the rocket when its engine burns out? What is its velocity at that point?

(b) The rocket continues upward after the engine burns out, subject only the downward pull of gravity. What is the maximum height that the rocket reaches? Remember, the acceleration due to gravity is 32 ft/s^2.

9. Sketch the region bounded by the indicated curves and find its area.

(a) $y = 2x - x^2$ and $y = x^2 - 4x$.

(b) $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi$.

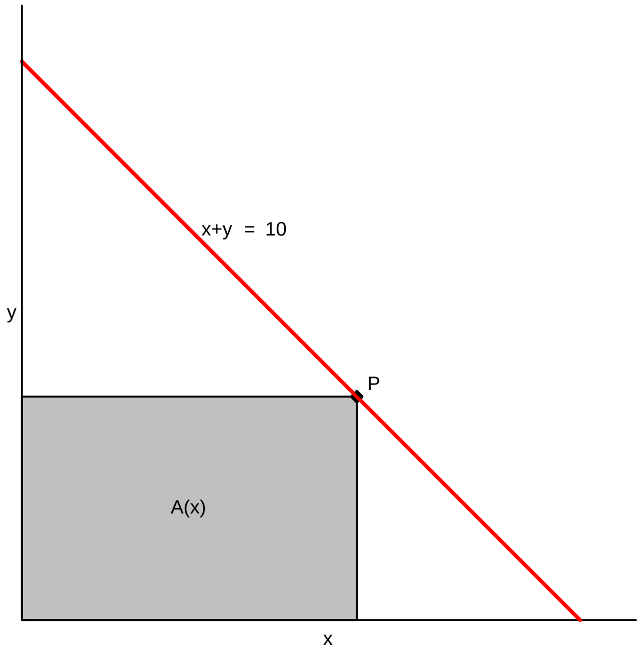
(c) $y = |x|$ and $y = x^3 - \frac{5}{3}x^2 - x + \frac{8}{3}$.

10. Consider a rectangle inscribed in the first-quadrant region that lies between the x-axis and the line $y = 10 - x$, as shown in the figure below.

(a) Express the area of this rectangle as a function $A(x)$ of the x-coordinate of its vertex P on the line.

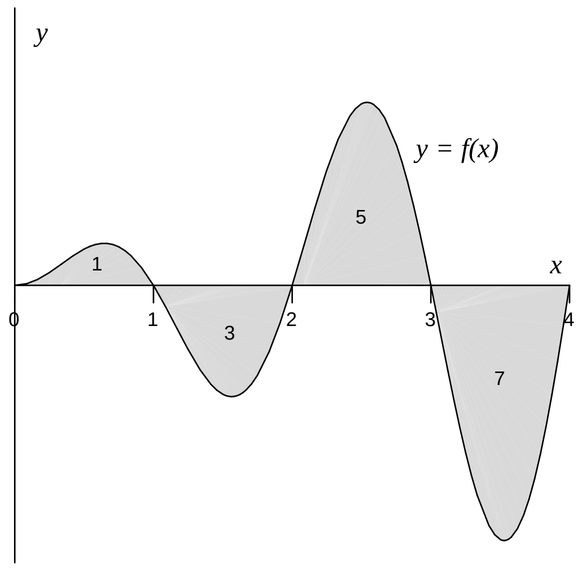
(b) Find the average value of $A(x)$.

(c) Find every value of x for which $A(x)$ attains its average value.



11. Consider the function $f(x)$ whose graph on the interval $[0, 4]$ is shown below. The numbers in each shaded region indicate the area of that region. Let

$$g(x) = \int_1^x f(t)\,dt.$$



(a) Compute $g(0)$, $g(1)$, $g(2)$, $g(3)$ and $g(4)$.

(b) Find and classify the critical points of $g(x)$.

(c) Find the absolute maximum and minimum values of $g(x)$ on the interval $[0, 4]$.

(d) Sketch the graph of $g(x)$.