Math 1312 Spring 2007

Calculus II

Second Midterm Exam

Thursday, March 1, 7:00 PM - 9:00 PM

Your name (please print):

Instructions: Other than a single 8.5” $\times$ 11” page of handwritten notes, this is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ______________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Points	10	10	10	10	10	10	10	10	10
Score									

Total:____________________

Problems 1 - 3: Solve the initial value problem. In your answer, be sure to express y as a function of x.

1.
$$y'' - 4y' + y = 0$$
$$y(0) = 2 - \sqrt{3}$$
$$y'(0) = 5$$

2.
$$4\frac{dy}{dx} - e^{-x/4} - 3y = 0$$
$$y(-1) = 0$$

3.
$$xy' + y = y^2$$
$$y(2) = 2$$

4. Find constants b and c so that $25y'' + by' + cy = 0$ has the general solution $y = e^{x/5}(c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x)$.

Problems 6 - 7: *Explain* why the integral is improper, then determine if it is convergent or divergent. Evaluate the integral if it is convergent.

5.
$$\int_1^9 (x-1)^{-2/3}\, dx$$

6.
$$\int_1^\infty \frac{\ln x}{x}\, dx$$

7. Recall that if we put a hole in the bottom of a tank filled with water, then the depth y of the water in the tank at time t satisfies the differential equation

$$A(y)\frac{dy}{dt} = -c\sqrt{y}$$

where $A(y)$ is the cross-sectional area of the tank at depth y and c is a constant related to gravity and the size of the hole. Suppose that a cylindrical tank is initially filled to a depth of 4 m and that the depth of water in the tank is 1 m two hours later. When will the tank be empty?

8. Consider a pond that initially contains 10 million gallons of fresh water. Water containing an undesirable chemical flows into the pond at the rate of 5 million gallons per year. The concentration $c(t)$ of chemical in the incoming water varies periodically with time according to the expression $c(t) = 2 + \sin 2t$ grams per gallon. Write down an initial value problem that models the amount of chemical in the pond. Be sure to *define* any variables you introduce and *specify* their units. *Do not solve this initial value problem!*

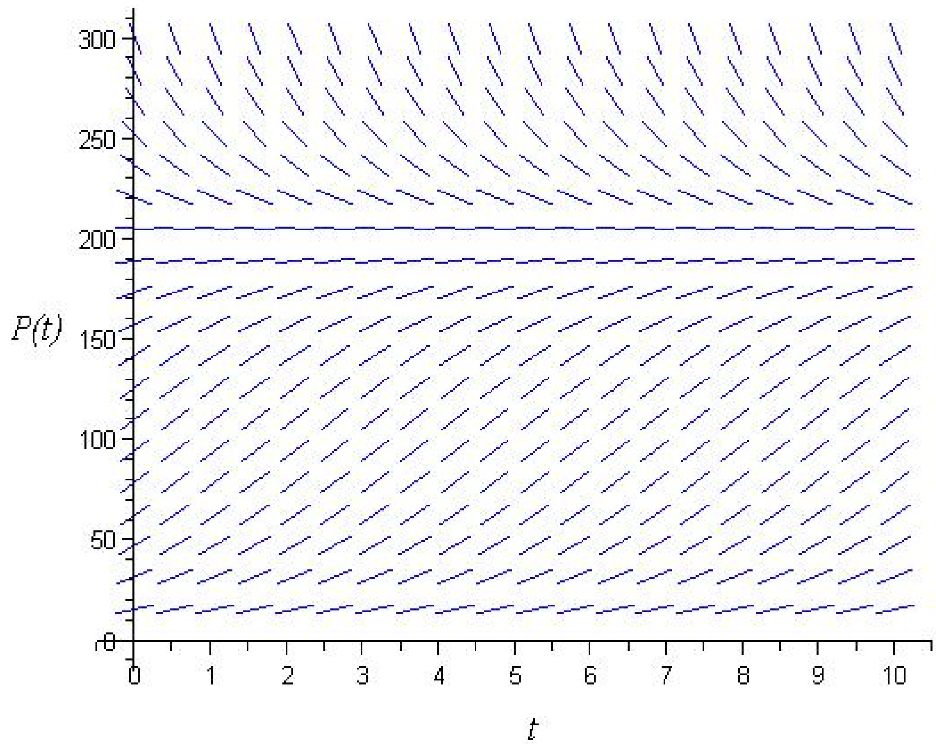
9. A certain population's growth can be modeled by the logistic equation

$$\frac{dP}{dt} = 0.6P - 0.003P^2$$

where P is in millions of individuals and t is in years.

a. Determine the size M of the population after a very long time (you *do not* need to solve for P to do this).

b. Use the slope field below to estimate how long it will take the population to reach 90% of the limiting population M you found in part (a), given that initially there are 10 million individuals.



Slope field for $\frac{dP}{dt} = 0.6P - 0.003P^2$