# Math 1312 Spring 2007

## Calculus II

## Final Exam

## Saturday, May 5, 6:30 PM - 9:30 PM

Your name (please print):

**Instructions:** This is a closed book exam, however you may use up to 4 pages of handwritten notes. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided, work each problem on a separate sheet of paper, and staple your pages in the correct order when you are finished.

**The Honor Code requires that you neither give nor receive any aid on this exam.**

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

**Pledged:** _____

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Do not write below this line

| Problem | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| Points | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Score | | | | | | | | |

| Problem | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **Total** |
|---|---|---|---|---|---|---|---|---|
| Points | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 150 |
| Score | | | | | | | | |

**1.** Find the Taylor polynomial of degree three about $a = \pi/2$ for $f(x) = \sin x$.

**2.** Find parametric equations for the line of intersection of the planes $x - y + 2z = 2$ and $2x + y - z = 1$.

**3.** Let $P = (1, 0, 2), Q = (-2, 1, 0), R = (1, 1, 1)$. Find the area of triangle $PQR$ as well as an equation for the plane containing this triangle.

**4.** Let

$$A = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & -1 & -2 \end{pmatrix}.$$

a. Show that

$$A^{-1} = \begin{pmatrix} -1 & 3 & 1 \\ 4 & -10 & -3 \\ -2 & 5 & 1 \end{pmatrix}.$$

b. Use part (a) to solve the system of equations

$$\begin{aligned} 5x + 2y + z &= 3 \\ 2x + y + z &= -2 \\ -y - 2z &= 1. \end{aligned}$$

**5.** Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 6 & 4 \end{pmatrix}, B = \begin{pmatrix} -3 & 1 \\ 2 & 0 \end{pmatrix}.$$

Compute whichever of the following that are defined: $AB$, $BA$, $AB - A$, $BA - A$.

**6.** Let $\mathbf{u} = \langle -3, 1, 0 \rangle$, $\mathbf{v} = \langle 0, -1, 2 \rangle$. Compute the following quantities.

a. $2\mathbf{u} - 3\mathbf{v}$

b. $\mathbf{u} \cdot \mathbf{v}$

c. $\mathbf{u} \times \mathbf{v}$

d. The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

**7.** Prove the *Pythagorean theorem:* if $\mathbf{a}$ and $\mathbf{b}$ are perpendicular vectors in $\mathbb{R}^3$ then

$$|\mathbf{a}|^2 + |\mathbf{b}|^2 = |\mathbf{a} + \mathbf{b}|^2.$$

[*Hint:* Remember that $|\mathbf{v}| = \mathbf{v} \cdot \mathbf{v}$.]

**Problems 8 - 10:** Evaluate the indefinite integral.

**8.** $\displaystyle\int \frac{2x+1}{x^3+x^2}\,dx$

**9.** $\displaystyle\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$

**10.** $\displaystyle\int \arctan x\,dx$

**Problems 11 - 13:** Find the general solution to the differential equation.

**11.** $9y'' - 6y' + 37y = 0$

**12.** $\displaystyle\frac{dy}{dx} = \frac{x^3+x}{2y}$

**13.** $\displaystyle xy' + 3y = \frac{1}{x}\ ,\ (x > 0)$

**Problems 14 - 15:** Determine if the series converges or diverges. If it converges, determine if the convergence is absolute or conditional.

**14.** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+4}$

**15.** $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$