Problems 1 through 4: Determine whether or not the improper integrals converge. Evaluate those that do converge.

1. $\displaystyle\int_{-1}^{4} \frac{dx}{4+3x-x^2}$

2. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^{3/2}}$

3. $\displaystyle\int_{0}^{2} x \ln x \, dx$

4. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x}{(\cos x)^{4/3}} \, dx$

Problems 5 through 8: Find the general solution to the differential equation. When given initial conditions, find the solution to initial value problem as well. In each case, be sure to solve for y in terms of x.

5. $4y'' - 2y' + y = 0$
$y(0) = -1$, $y'(0) = 2$

6. $x^2 y' + xy = 1$ $(x > 0)$
$y(1) = 2$

7. $2y'' + 8y' - y = 0$

8. $\dfrac{dy}{dx} = \dfrac{y^2 - 2y - 3}{x}$

9. Consider the second order differential equation

$$3y'' + 2y' = 5e^{5x}.$$

Find the general solution as follows: antidifferentiate both sides and solve the resulting first order equation. Your answer should involve *two* arbitrary constants.

10. A tank contains 100 L of water. A solution with a salt concentration of 0.4 kg/L is added at a rate of 5 L/min. The solution is kept mixed and is drained from the tank at a rate of 3 L/min. Let $y(t)$ be the amount of salt (in kilograms) after t minutes.

a. Show that y satisfies the differential equation

$$\frac{dy}{dt} = 2 - \frac{3y}{100+2t}.$$

b. Solve the differential equation in part (a) and find the concentration of salt in the tank after 20 minutes.

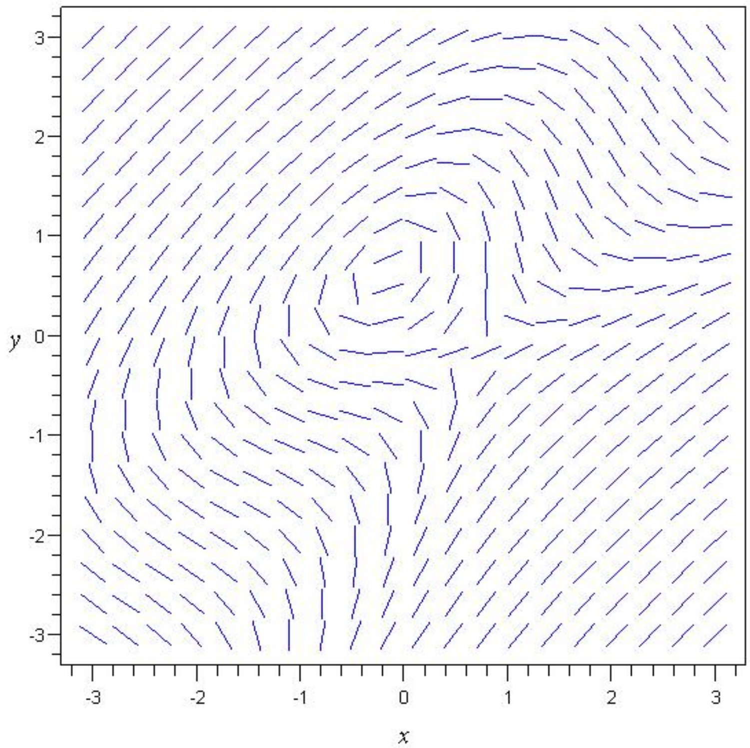
11. The diagram below shows the slope field for the differential equation

$$\frac{dy}{dx} = F(x, y). \tag{1}$$

a. Use the slope field to sketch the solution $y(x)$ to (1) that satisfies $y(-1) = 1$. Use your sketch to estimate $y(1)$.

b. Use the slope field to sketch the solution $y(x)$ to (1) that satisfies $y(2) = 1$. Use your sketch to estimate $y(-1)$.

c. Use the slope field to sketch the solution $y(x)$ to (1) that satisfies $y(0) = -1$. Is it possible to use your sketch to estimate $y(2)$? Why or why not?



Slope field for Problem 11.

12. Consider a population of individuals whose birth rate at any given time is proportional to the current population and whose death rate is *inversely* proportional to the current population.

a. If $P(t)$ is the size of the population at time t, show that P satisfies the differential equation

$$\frac{dP}{dt} = kP^2 - l$$

for some positive constants k and l.

b. If $l = 25$ and $k = 0.01$, solve the differential equation in part (a).

c. If the initially there are 10 individuals, what happens to the population in the long run? What if there are 60 individuals initially?