1.Evaluate the limit

$$\lim_{n\to\infty} \sqrt[n]{n^2+1}$$

2. Find the limit of the sequence

$$\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\}$$

3. Let $\{F_n\}$ denote the Fibonacci sequence, which is defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. That is, the next term in the Fibonacci sequence is the sum of the preceding two terms:

$$\{F_n\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots\}.$$

Show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{F_n}$$

converges absolutely. [*Hint:* $\lim_{n\to\infty} F_{n+1}/F_n = (1+\sqrt{5})/2$.]

4. Find a power series representation for the function

$$f(x) = \frac{1}{3-x}$$

in powers of $x+3$. Be sure to state where (for what values of x) this representation is valid.

5. Let

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

a. Show that the series has radius of convergence 1 and determine the interval of convergence.

b. Show that

$$f'(x) = \frac{1}{1-x}$$

for all $|x| < 1$.

c. Use part (b) to conclude that

$$f(x) = \ln(1-x)$$

for $|x| < 1$.

d. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}.$$

6. Use the integral test to show that the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges when $p > 1$.

7. The terms of the series

$$\sum_{n=1}^{\infty} (-1)^n a_n$$

are defined recursively by the equation

$$a_1 = 2 a_{n+1} = \frac{5n+1}{4n+3} a_n.$$

Determine whether the series converges or diverges.

8. Determine the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x+5)^n}{(n+1)n!}.$$