Example 1:

$$\begin{array}{rcrcr} x_1 & + & 5x_2 & = & 7 \\ -2x_1 & - & 7x_2 & = & -5 \end{array} \qquad \begin{pmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{pmatrix}$$

$$\begin{array}{rcrcr} x_1 & + & 5x_2 & = & 7 \\ & & 3x_2 & = & 9 \end{array} \qquad \begin{pmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{pmatrix}$$

$$\begin{array}{rcrcr} x_1 & + & 5x_2 & = & 7 \\ & & x_2 & = & 3 \end{array} \qquad \begin{pmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\begin{array}{rrcr} x_1 & & = & -8 \\ & x_2 & = & 3 \end{array} \qquad \begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{pmatrix}$$

Example 2:

$$\begin{array}{rcrcrcr} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \begin{pmatrix} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{pmatrix}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{pmatrix}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ & - & x_2 & - & x_3 & = & 3 \end{array} \qquad \begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & 2x_2 & + & x_3 & = & -8 \end{array} \qquad \begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & -8 \end{pmatrix}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array} \qquad \begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{pmatrix}$$

$$
\begin{array}{rcrcrcr}
x_1 & - & 2x_2 & & & = & 6 \\
 & - & x_2 & & & = & 5 \\
 & & & - & x_3 & = & -2
\end{array}
\qquad
\begin{pmatrix}
1 & -2 & 0 & 6 \\
0 & -1 & 0 & 5 \\
0 & 0 & -1 & -2
\end{pmatrix}
$$

$$
\begin{array}{rcrcrcr}
x_1 & & & & & = & -4 \\
 & - & x_2 & & & = & 5 \\
 & & & - & x_3 & = & -2
\end{array}
\qquad
\begin{pmatrix}
1 & 0 & 0 & -4 \\
0 & -1 & 0 & 5 \\
0 & 0 & -1 & -2
\end{pmatrix}
$$

$$
\begin{array}{rrrcr}
x_1 & & & = & -4 \\
 & x_2 & & = & -5 \\
 & & x_3 & = & 2
\end{array}
\qquad
\begin{pmatrix}
1 & 0 & 0 & -4 \\
0 & 1 & 0 & -5 \\
0 & 0 & 1 & 2
\end{pmatrix}
$$

EXAMPLE 3:

$$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & + & 3x_3 & = & -5 \\ x_1 & + & 2x_2 & + & 8x_3 & = & 3 \\ x_1 & & & + & 2x_3 & = & -1 \end{array} \qquad \begin{pmatrix} 3 & -1 & 3 & -5 \\ 1 & 2 & 8 & 3 \\ 1 & 0 & 2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 1 & 2 & 8 & 3 \\ 3 & -1 & 3 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 2 & 6 & 4 \\ 0 & -1 & -3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{array}{rcrcrcr} x_1 & & & + & 2x_3 & = & -1 \\ & & x_2 & + & 3x_3 & = & 2 \\ & & & & 0 & = & 0 \end{array}$$

Any choice of x_3 yields a solution by setting

$$\begin{aligned} x_1 &= -2x_3 - 1 \\ x_2 &= -3x_3 + 2. \end{aligned}$$

Example 4:

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \begin{pmatrix} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ & - & 2x_2 & - & 6x_3 & = & 10 \\ & & & & 0 & = & -1 \end{array}$$

Since the equation

$$0 = -1$$

is never true, this system has no solutions.