Problem 1. If **a**, **b** and **c** are nonzero vectors and $\mathbf{c} = |\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$, show that **c** bisects the angle between **a** and **b**.

Problem 2. Find the volume of the tetrahedron with vertices $(-4, -5, 2)$, $(-2, 1, 3)$, $(0, 3, -4)$ and $(0, -2, 2)$.

Problem 3. Find the line of intersection of the two planes $x+3y+z = 4$ and $2x+4y+z = -1$.

Problem 4.

a. Find the point where the lines

$$\begin{aligned} \mathbf{r}_1(t) &= \langle -4t, -5 + 3t, -3 - 2t\rangle \\ \mathbf{r}_2(t) &= \langle 6 - 5t, -t, -5\rangle \end{aligned}$$

intersect.

b. Find an equation for the plane containing these lines. Write your answer in the form $ax + by + cz + d = 0$.

Problem 5. Find parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

Problem 6. Find a vector function that represents the intersection of the surfaces $x^2+y^2 = 4$ and $z = xy$.

Problem 7. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t\rangle$, $0 \le t \le 1$.

Problem 8. Draw a contour map of the function $f(x, y) = (y - 2x)^2$ and use this to sketch the graph of $z = f(x, y)$.

Problem 9. Evaluate the following limits, or show that they do not exist.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{2x^2 + 3y^2}}$

b. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{2x^3+3y^6}}$

c. $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{2xyz^2}{1-x-y-z}$

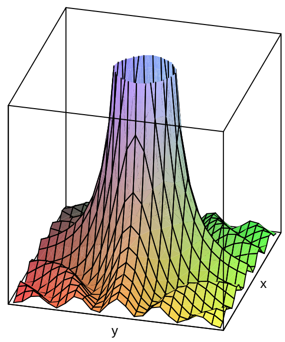
Problem 10. Verify that the function $z = \ln(e^x + e^y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0.$$

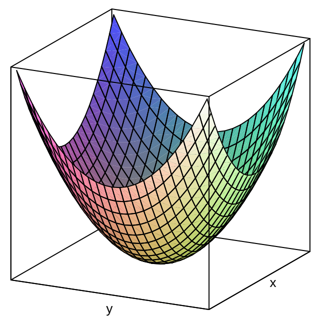
Problem 11. Match the following functions with their graphs and contour maps (shown on the following page).

a. $\ln\left(x^2+y^2\right)$ **b.** $2\left(x^2+y^2\right)-5$

c. $\dfrac{1+\cos(xy)}{x^2+y^2}$ **d.** y^2-x^3



(A)



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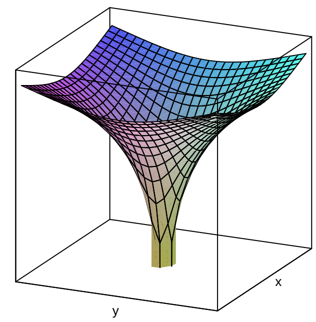

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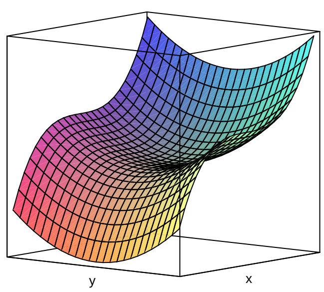

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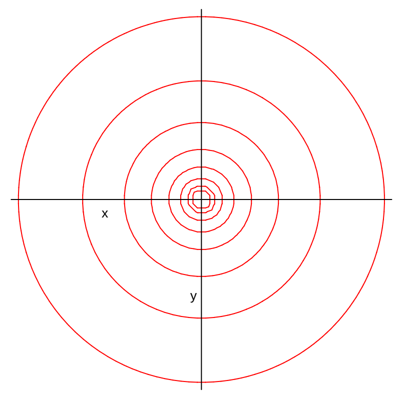
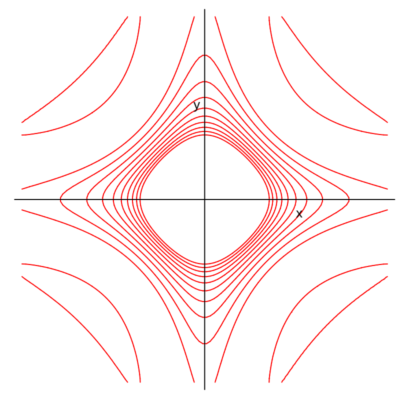


(i)



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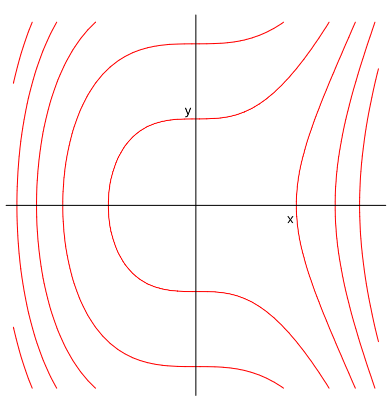

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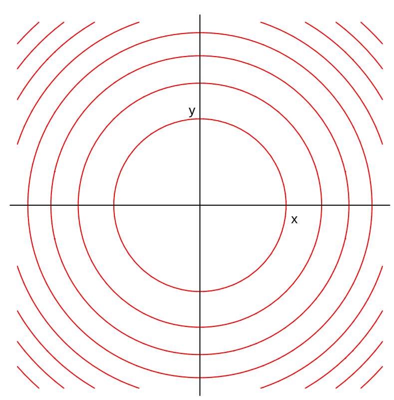

(iv)