Problem 1. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are nonzero vectors and $\mathbf{c} = |\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$, show that $\mathbf{c}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$.

Problem 2. Find the volume of the tetrahedron with vertices $(-4, -5, 2)$, $(-2, 1, 3)$, $(0, 3, -4)$ and $(0, -2, 2)$.

Problem 3. Find the line of intersection of the two planes $x+3y+z = 4$ and $2x+y+z = -1$.

Problem 4.

a. Find the point where the lines

\[
\mathbf{r}_1(t) = \langle -4t, -5 + 3t, -3 - 2t \rangle \\
\mathbf{r}_2(t) = \langle 6 - 5t, -t, -5 \rangle
\]

intersect.

b. Find an equation for the plane containing these lines. Write your answer in the form $ax + by + cz + d = 0$.

Problem 5. Find parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

Problem 6. Find a vector function that represents the intersection of the surfaces $x^2+y^2 = 4$ and $z = xy$.

Problem 7. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$, $0 \leq t \leq 1$.

Problem 8. Draw a contour map of the function $f(x, y) = (y - 2x)^2$ and use this to sketch the graph of $z = f(x, y)$.

Problem 9. Evaluate the following limits, or show that they do not exist.

a. $\lim_{{(x,y) \to (0,0)}} \frac{xy}{\sqrt{2x^2 + 3y^2}}$
b. \( \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{2x^3 + 3y^6}} \)

c. \( \lim_{(x,y,z) \to (1,1,1)} \frac{2xyz^2}{1 - x - y - z} \)

**Problem 10.** Verify that the function \( z = \ln(e^x + e^y) \) satisfies the partial differential equation

\[
\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left( \frac{\partial^2 z}{\partial x \partial y} \right)^2 = 0.
\]

**Problem 11.** Match the following functions with their graphs and contour maps (shown on the following page).

a. \( \ln(x^2 + y^2) \)  
   b. \( 2(x^2 + y^2) - 5 \)

c. \( \frac{1 + \cos(xy)}{x^2 + y^2} \)  
   d. \( y^2 - x^3 \)