Problem 1. If $z = f(x, y)$, where f is differentiable, and

$$\begin{array}{rclcrcl} x & = & g(t) & & y & = & h(t) \\ g(3) & = & 4 & & h(3) & = & 5 \\ g'(3) & = & 5 & & h'(3) & = & -4 \\ f_x(4,5) & = & 6 & & f_x(3,3) & = & 1 \\ f_y(4,5) & = & -8 & & f_y(3,3) & = & 10 \end{array}$$

find dz/dt when $t = 3$.

Problem 2. If $R = \ln(u^2 + v^2 + w^2)$ and $u = x + 2y$, $v = 2x - y$ and $w = 2xy$, compute $\partial R/\partial x$ and $\partial R/\partial y$ when $x = y = 1$.

Problem 3. The tangent line to the level curve $f(x, y) = 5$ at the point $(1, 2)$ has the equation $3x - 4y = -5$. Find all the possible directions of $\nabla f(1, 2)$. Specify your answers as unit vectors.

Problem 4. The plane $4x - 3y + 8z = 40$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse. Find an equation for the tangent line to this ellipse at the point $(3, 4, 5)$.

Problem 5. Find all points at which the function $f(x, y) = x^2 + y^2 - 2x - 4y$ increases most rapidly in the direction of $\mathbf{i} + \mathbf{j}$.

Problem 6. Let $\mathbf{u} = \langle 3/5, 4/5\rangle$ and $\mathbf{v} = \langle 5/13, -12/13\rangle$. Suppose f is differentiable at (a, b) and that $D_{\mathbf{u}} f(a, b) = 2$ and $D_{\mathbf{v}} f(a, b) = -1$. Find $f_x(a, b)$ and $f_y(a, b)$.

Problem 7. Find and classify the critical points of the function $f(x, y) = x^3 - 12xy + 8y^3$.

Problem 8. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to the constraint $2x + 3y = 6$.

Problem 9.

a. Find the absolute maximum value of the function $f(x, y, z) = (xyz)^{1/3}$ given that x, y and z are nonnegative numbers satisfying $x + y + z = 1$. Carefully justify your answer.

b. Use the result of part (**a**) to deduce the *arithmetic-geometric mean inequality*: if $a, b, c \geq 0$ then

$$\sqrt[3]{abc} \leq \frac{a+b+c}{3}.$$

When can equality hold?

Problem 10. Find the absolute maximum and minimum values of $f(x, y) = e^{-xy}$ on the region $\{(x, y) \mid x^2 + 4y^2 \leq 2\}$.

Problem 11. Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = [0, 1] \times [-2, 3]$.