Problem 1. If \( z = f(x, y) \), where \( f \) is differentiable, and

\[
\begin{align*}
  x &= g(t) & y &= h(t) \\
  g(3) &= 4 & h(3) &= 5 \\
  g'(3) &= 5 & h'(3) &= -4 \\
  f_x(4,5) &= 6 & f_x(3,3) &= 1 \\
  f_y(4,5) &= -8 & f_y(3,3) &= 10
\end{align*}
\]

find \( dz/dt \) when \( t = 3 \).

Problem 2. If \( R = \ln(u^2 + v^2 + w^2) \) and \( u = x + 2y, v = 2x - y \) and \( w = 2xy \), compute \( \partial R/\partial x \) and \( \partial R/\partial y \) when \( x = y = 1 \).

Problem 3. The tangent line to the level curve \( f(x, y) = 5 \) at the point \( (1, 2) \) has the equation \( 3x - 4y = -5 \). Find all the possible directions of \( \nabla f(1, 2) \). Specify your answers as unit vectors.

Problem 4. The plane \( 4x - 3y + 8z = 40 \) intersects the cone \( z^2 = x^2 + y^2 \) in an ellipse. Find an equation for the tangent line to this ellipse at the point \( (3, 4, 5) \).

Problem 5. Find all points at which the function \( f(x, y) = x^2 + y^2 - 2x - 4y \) increases most rapidly in the direction of \( \mathbf{i} + \mathbf{j} \).

Problem 6. Let \( \mathbf{u} = \langle 3/5, 4/5 \rangle \) and \( \mathbf{v} = \langle 5/13, -12/13 \rangle \). Suppose \( f \) is differentiable at \( (a, b) \) and that \( D_u f(a, b) = 2 \) and \( D_v f(a, b) = -1 \). Find \( f_x(a, b) \) and \( f_y(a, b) \).

Problem 7. Find and classify the critical points of the function \( f(x, y) = x^3 - 12xy + 8y^3 \).

Problem 8. Find the absolute maximum and minimum values of \( f(x, y) = x^2 + y^2 \) subject to the constraint \( 2x + 3y = 6 \).

Problem 9.

a. Find the absolute maximum value of the function \( f(x, y, z) = (xyz)^{1/3} \) given that \( x, y \) and \( z \) are nonnegative numbers satisfying \( x + y + z = 1 \). Carefully justify your answer.
b. Use the result of part (a) to deduce the arithmetic-geometric mean inequality: if \( a, b, c \geq 0 \) then

\[
\sqrt[3]{abc} \leq \frac{a + b + c}{3}.
\]

When can equality hold?

Problem 10. Find the absolute maximum and minimum values of \( f(x, y) = e^{-xy} \) on the region \( \{(x, y) | x^2 + 4y^2 \leq 2\} \).

Problem 11. Find the volume of the solid that lies under the plane \( 3x + 2y + z = 12 \) and above the rectangle \( R = [0, 1] \times [-2, 3] \).