Calculus III
Spring 2009

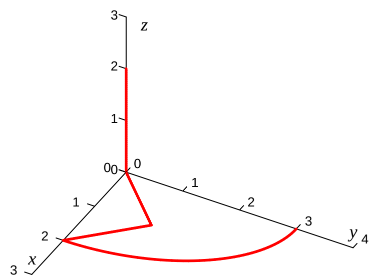
Final Exam Practice Problems

Problem 1. Find the average value of the x-coordinate on the quarter of the circle $x^2+y^2=1$ that lies in the first octant.

Problem 2. Let

$$\mathbf{F} = (3x^2yz - 3y)\mathbf{i} + (x^3z - 3x)\mathbf{j} + (x^3y + 2z)\mathbf{k}.$$

Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve from $(0,0,2)$ to $(0,3,0)$ shown below.



Problem 3. Evaluate

$$\int_C (e^x - xy)\ dx + \left(xy + \frac{x^2}{2} + \sin y\right)\ dy$$

where C is the triangle with vertices $(0,1)$, $(-1,0)$ and $(2,0)$, traveled clockwise.

Problem 4. Show that the average distance from a point on the sphere $x^2+y^2+z^2=a^2$ to its north pole is $4a/3$.

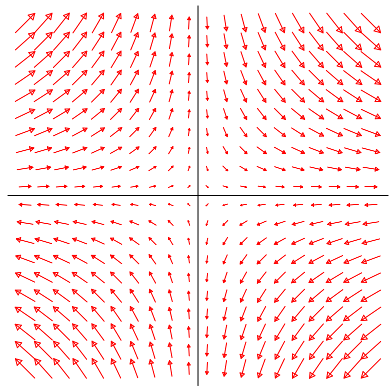
Problem 5.

a. Find an equation of the tangent plane at the point $(4, -2\sqrt{2}, 1)$ to the parametric surface S given by

$$\mathbf{r}(u,v) = v^2\mathbf{i} - \sqrt{2}uv\mathbf{j} + u^2\mathbf{k} \quad 0 \le u \le 3,\ -3 \le v \le 3.$$

b. Find the surface area of S.

Problem 6. Explain why the vector field shown below is *not* conservative.



Problem 7. Let $\mathbf{r} = \langle x, y, z \rangle$ and $\mathbf{F}(\mathbf{r}) = |\mathbf{r}|^2 \mathbf{r}$. If S is the surface $x^2 + y^2 + z^2 = a^2$, find the flux of $\mathbf{F}$ through S without actually performing an antidifferentiation. [*Hint:* Find a simple description of the normal vector to S *without* parametrizing it.]

Problem 8. If $\mathbf{a}$ is a constant vector, $\mathbf{r} = \langle x, y, z \rangle$, and S is an oriented surface with simple, closed, positively oriented boundary curve C, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}.$$

Problem 9. Evaluate

$$\int_C (y + \sin x)\, dx + (z^2 + \cos y)\, dy + x^3\, dz$$

where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \le t \le 2\pi$. [*Hint:* Observe that C lies on the surface $z = 2xy$.]

Problem 10. Calculate the flux of the vector field $\mathbf{F}(x, y, z) = \langle x^4, -x^3 z^2, 4xy^2 z \rangle$ across the boundary of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2$ and $z = 0$.