Problem 1. Find the average value of the \( x \)-coordinate on the quarter of the circle \( x^2 + y^2 = 1 \) that lies in the first octant.

Problem 2. Let
\[
\mathbf{F} = (3x^2yz - 3y)\mathbf{i} + (x^3z - 3x)\mathbf{j} + (x^3y + 2z)\mathbf{k}.
\]
Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the curve from \((0, 0, 2)\) to \((0, 3, 0)\) shown below.

Problem 3. Evaluate
\[
\int_C (e^x - xy) \, dx + \left( xy + \frac{x^2}{2} + \sin y \right) \, dy
\]
where \( C \) is the triangle with vertices \((0, 1), (-1, 0)\) and \((2, 0)\), traveled clockwise.

Problem 4. Show that the average distance from a point on the sphere \( x^2 + y^2 + z^2 = a^2 \) to its north pole is \( 4a/3 \).

Problem 5.

a. Find an equation of the tangent plane at the point \((4, -2\sqrt{2}, 1)\) to the parametric surface \( S \) given by
\[
\mathbf{r}(u, v) = v^2\mathbf{i} - \sqrt{2}uv\mathbf{j} + u^2\mathbf{k} \quad 0 \leq u \leq 3, \quad -3 \leq v \leq 3.
\]

b. Find the surface area of \( S \).
Problem 6. Explain why the vector field shown below is not conservative.

Problem 7. Let $\mathbf{r} = \langle x, y, z \rangle$ and $\mathbf{F}(\mathbf{r}) = |\mathbf{r}|^2 \mathbf{r}$. If $S$ is the surface $x^2 + y^2 + z^2 = a^2$, find the flux of $\mathbf{F}$ through $S$ without actually performing an antidifferentiation. [Hint: Find a simple description of the normal vector to $S$ without parametrizing it.]

Problem 8. If $\mathbf{a}$ is a constant vector, $\mathbf{r} = \langle x, y, z \rangle$, and $S$ is an oriented surface with simple, closed, positively oriented boundary curve $C$, show that

$$\int \int_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}.$$ 

Problem 9. Evaluate

$$\int_C (y + \sin x) \, dx + (z^2 + \cos y) \, dy + x^3 \, dz$$

where $C$ is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \leq t \leq 2\pi$. [Hint: Observe that $C$ lies on the surface $z = 2xy$.]

Problem 10. Calculate the flux of the vector field $\mathbf{F}(x, y, z) = \langle x^4, -x^3z^2, 4xy^2z \rangle$ across the boundary of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2$ and $z = 0$. 
