In exercises 1 - 3 you are given a region R in the xy-plane, a region S in the uv-plane, and a transformation that maps S to R. Sketch both regions and use the transformation to compute $\iint_R f(x,y)\,dA$ for the indicated function $f(x,y)$.

Exercise 1. R is the parallelogram with vertices $(0,0)$, $(4,1)$, $(1,2)$ and $(5,3)$; $S = [0,1] \times [0,1]$; the transformation is $x = 4u + v$, $y = u + 2v$; $f(x,y) = x + y^2$.

Exercise 2. R is the triangle with vertices $(2,1)$, $(3,4)$ and $(1,2)$; S is the triangle with vertices $(0,3)$, $(3,3)$ and $(2,7)$; the transformation is $x = (u+v)/3$, $y = (2v-u)/3$; $f(x,y) = xy$.

Exercise 3. R is the region in the first quadrant bounded by the curves $y = x$, $y = 4x$, $y = 1/x$ and $y = 4/x$; $S = [1,2] \times [1,2]$; the transformation is $x = u/v$, $y = uv$; $f(x,y) = \sqrt{y/x}\, e^{\sqrt{xy}}$.

Exercise 4. Let E denote the region enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

a. Find the area of E using a double integral with respect to x and y.

b. Show that the transformation $x = au$, $y = bv$ maps the disk $u^2 + v^2 \leq 1$ to the region E.

c. Use part (b) to compute the area of E by converting the integral from part (a) to uv-coordinates. Which method of computation is easier?