CALCULUS III
SPRING 2010

EXAM 1
PRACTICE PROBLEMS

Exercise 1. Consider the following four planes:

$$\begin{aligned} P_1: &\quad -4x - 2y + 2z = 4 \\ P_2: &\quad 5x + 2z = 5 \\ P_3: &\quad x - 4y + 3z = -1 \\ P_4: &\quad 6x + 3y - 3z = -6 \end{aligned}$$

Which are parallel? Are any of them identical? For those pairs that are not parallel, determine their lines of intersection.

Exercise 2. Are the points $(1, -1, -2)$, $(2, 4, -4)$ and $(5, 0, 2)$ colinear? If so, determine the line that contains them. If not, determine the plane that contains them, find the area of the triangle they form and determine if that triangle is right-angled.

Exercise 3. Do the lines $\ell_1(s) = \langle -1 + s, 4 + s, 1 + 3s\rangle$ and $\ell_2(t) = \langle -6t, 1 + 9t, -3t\rangle$ intersect? If so, find their point of intersection. If not, determine if they are skew or parallel and find the distance between them.

Exercise 4. Find the acute angle between two diagonals of a cube.

Exercise 5. Suppose that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 2$. Find $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$, $\mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$, $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w})$ and $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$.

Exercise 6. Parameterize the curve of intersection of the cylinder $y^2 + 4z^2 = 1$ and the plane $x + 2y + 3z = 0$. Find the tangent lines to this curve at the points $(-2, 1, 0)$ and $(-3/2, 0, 1/2)$ and the point where these lines intersect.

Exercise 7. Let $f(x, y) = \sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2}$. Describe and sketch the contours $g(x, y) = k$. What happens as $k \to \infty$? Use this information to sketch the graph of g.

Exercise 8. Evaluate the following limits, or show that they do not exist.

a. $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + 3y^2}$

b. $\lim_{(x,y,z)\to(0,0,0)} \frac{2x^2 + 2y^2 + z^2}{x^2 + 3y^2 + z^2}$

c. $\lim_{(x,y)\to(0,0)} \frac{\cos(y)\ln(x+2)}{x^4 + y^2 + 2}$

d. $\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$ [*Hint:* Substitute $r = \sqrt{x^2 + y^2}$. What happens to r as $(x, y) \to (0, 0)$?]