Math 2321 Spring 2009

Calculus III

First Midterm Exam

Friday, February 6

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Points	14	10	10	10	16	10	10	20
Score								

Total:_____

1. Find equations for the planes described below. Express your answers in the form $ax + by + cz + d = 0$.

a. The plane passing through the points $(-4, -5, 2)$, $(-2, 1, 3)$ and $(0, 3, -4)$.

b. A plane parallel to $x + 2y + 2z = 3$, whose distance to this plane is 2 units.

2. Find parametric equations for the tangent line to the curve with parametric equations

$$\begin{aligned} x &= 1 + 2\sqrt{t} \\ y &= t^3 - t \\ z &= t^3 + t \end{aligned}$$

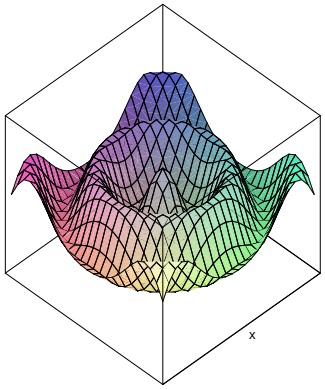
at the point $(3, 0, 2)$.

3. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j} + \sqrt{t}\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} + \mathbf{j}$.

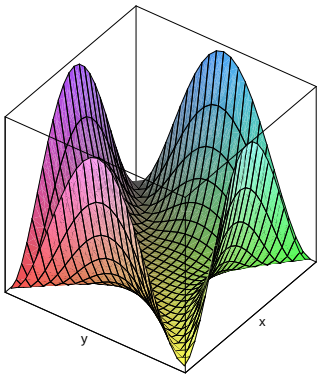
4. If $\mathbf{a}$ and $\mathbf{b}$ are vectors, show that $|\mathbf{a}+\mathbf{b}|^2 + |\mathbf{a}-\mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$.

5. Match the following functions with their graphs and contour maps (shown on the following page). You do not need to justify your answers.

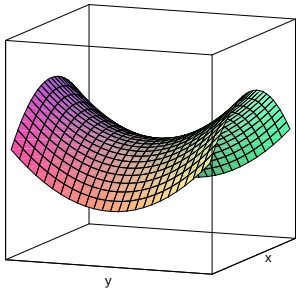
Function	Graph	Contour Map
$\dfrac{1}{1+x^2+y^2}$		
$(x^2-y^2)^2$		
$\cos\sqrt{y^2+x^2}$		
y^2-x^2		



(A)



(B)



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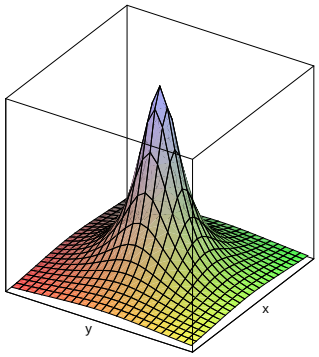
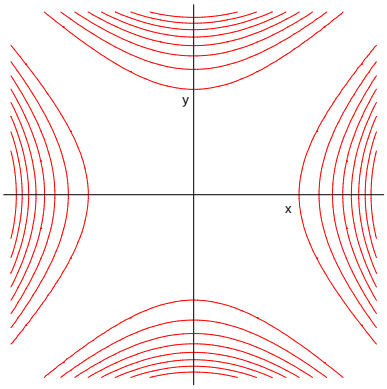
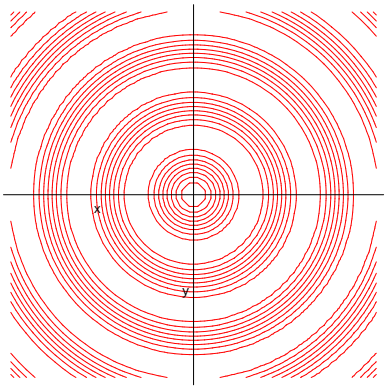


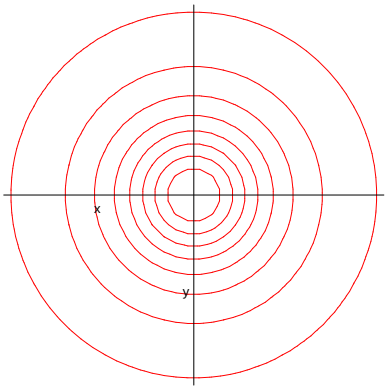
(D)



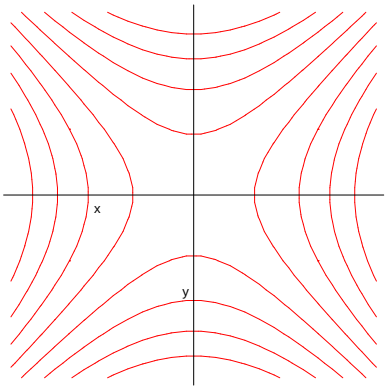
(I)



(II)



(III)



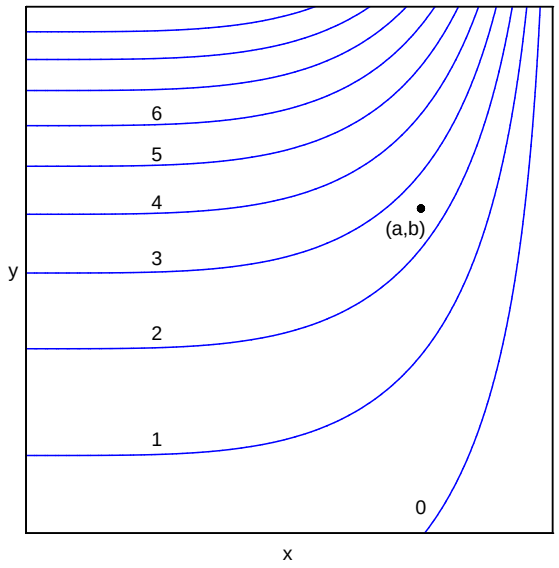
(IV)

6.

a. If $z = u\sqrt{v - w}$, find $\dfrac{\partial^3 z}{\partial u\, \partial v\, \partial w}$.

b. If $u = r \ln(rs^2t^3)$, find $\dfrac{\partial^3 u}{\partial r\, \partial s^2}$.

7. The diagram below shows the contour map of a function $f(x, y)$. Use the diagram to determine the sign of the indicated partial derivatives of f. You do not need to justify your answers.



a. $f_x(a, b)$ _____

b. $f_y(a, b)$ _____

c. $f_{xx}(a, b)$ _____

d. $f_{yy}(a, b)$ _____

e. $f_{xy}(a, b)$ _____

8. Let

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

a. Show that f is continuous at $(0, 0)$.

b. Compute $f_x(x, y)$ for $(x, y) \neq (0, 0)$.

c. Compute $f_x(0, 0)$ *using the limit definition of the partial derivative.*

d. Show that $f_x(x, y)$ is *not* continuous at $(0, 0)$.