Calculus III
Spring 2010

Exam 2
Practice Problems

Exercise 1. Verify that the function $z = \ln(e^x + e^y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0.$$

Exercise 2. Explain why the function $f(x, y) = \sqrt{x + e^{4y}}$ is differentiable at the point $(3, 0)$. Then use a linear approximation to estimate $f(2.91, 0.06)$.

Exercise 3. Recall that if $L(x, y)$ is the linear approximation to $f(x, y)$ at the point (a, b) then f is said to be *differentiable* at (a, b) provided

$$\lim_{(x,y)\to(a,b)} \frac{f(x, y) - L(x, y)}{\sqrt{(x-a)^2 + (y-b)^2}} = 0.$$

Use this definition to prove that $f(x, y) = x^2 + xy$ is differentiable at any point (a, b).

Exercise 4. If $u = x^2 + yz$, $x = pr\cos\theta$, $y = pr\sin\theta$ and $z = p + r$ find $\dfrac{\partial u}{\partial p}, \dfrac{\partial u}{\partial r}$ and $\dfrac{\partial u}{\partial \theta}$ when $p = 2$, $r = 3$ and $\theta = 0$.

Exercise 5. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

Exercise 6. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

Exercise 7. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $y = x^2 + z^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(-1, 2, 1)$.

Exercise 8. Find and classify the critical points of $f(x, y) = xy + \ln x + y^2$.

Exercise 9. Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region in the xy-plane with vertices $(0, 0)$, $(6, 0)$ and $(0, 6)$.

Exercise 10. Find the absolute maximum and minimum values of $f(x, y, z) = xyz$ on the sphere of radius 3 centered at the origin. Where do these values occur.

Exercise 11. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?