# Math 2321 Spring 2009 <br> Calculus III 

Second Midterm Exam<br>Wednesday, March 4

Your name (please print):

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. Except on multiple choice questions, you must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.
Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: $\qquad$

Do not write below this line ${ }^{1}$

| Problem | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points | 5 | 5 | 5 | 5 | 10 | 10 | 10 | 10 |
| Score |  |  |  |  |  |  |  |  |

Total: $\qquad$

## Section 1: Multiple choice.

1. The directional derivative of $f(x, y, z)=x y+y z+x z$ at $(1,3,1)$ in the direction of $(4,1,7)$ is:
a. 6
b. $32 / 7$
c. 32
d. $46 / \sqrt{66}$
2. The steepest slope on the graph of $f(x, y)=y^{2} / x$ at the point $(2,4,8)$ is:
a. $4 \sqrt{2}$
b. 4
c. $\quad-4$
d. $2 \sqrt{3}$
3. The tangent plane to the surface $\sqrt{x}+\sqrt{y}+\sqrt{z}=4$ at the point $(1,4,1)$ is given by:
a. $2 x+y+z=7$
b. $2 x+y+2 z=0$
c. $2 x+y+2 z=8$
d. $x+2 y+z=10$
4. The function $g(x, y)=4 x y-x^{2} y-y^{2} x$ has critical points at $(4,0)$ and $(4 / 3,4 / 3)$. In fact:
a. Both are saddle points of $g$
b. $(4,0)$ is a local maximum of $g$,
c. $(4,0)$ is a saddle point of $g$, $(4 / 3,4 / 3)$ is a local minimum of $g$
d. $(4,0)$ is a saddle point of $g$, $(4 / 3,4 / 3)$ is a local maximum of $g$

## Section 2: Free response

5. The temperature at each point $(x, y)$ is given by $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after $t$ seconds is given by $x=\sqrt{1+t}, y=2+\frac{1}{3} t$, where $x$ and $y$ are measured in centimeters. The temperature function satisfies $T_{x}(2,3)=4$ and $T_{y}(2,3)=3$. How fast is the temperature rising on the bug's path after 3 seconds?
6. Find the absolute maximum and minimum values of $f(x, y, z)=x^{2}+8 y+6 z$ if $(x, y, z)$ is restricted to lie on the surface of the sphere of radius 5 centered at the origin.
7. Let $f(x, y)=(1+x-y)^{50}$.
a. Find the linear approximation to $f(x, y)$ at $(0,0)$.
b. Is $f(x, y)$ differentiable at $(0,0)$ ? Justify your answer.
c. Estimate $f(0.01,0.02)$.
8. Evaluate $\int_{0}^{\pi} \int_{0}^{1} y \cos x y d y d x$.
