Calculus III
Spring 2010

Exam 3 Practice Problems

Problem 1. Evaluate the following integrals.

a. $\displaystyle\iint_R xy\,dA$ where R is the region bounded by the curves $x = y^2$ and $y = x - 2$.

b. $\displaystyle\int_1^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy$ [*Hint:* Reverse the order of integration.]

c. $\displaystyle\iint_R x\,dA$ where R is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

d. $\displaystyle\iiint_R xy\,dV$ where R is the solid tetrahedron with vertices $(0,0,0), (1/3,0,0), (0,1,0)$ and $(0,0,1)$.

e. $\displaystyle\iiint_H z^3\sqrt{x^2+y^2+z^2}\,dV$ where H is the solid hemisphere that lies above the xy-plane and has center $(0,0,0)$ and radius 1.

f. $\displaystyle\int_C y\,dx + (x + y^2)\,dy$ where C is the ellipse $4x^2 + 9y^2 = 36$ with counterclockwise orientation.

g. $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = xy\mathbf{i} + x^2\mathbf{j}$ and C is the curve given by $\mathbf{r}(\mathbf{t}) = \sin t\mathbf{i} + (1+t)\mathbf{j}$, $0 \le t \le \pi$.

Problem 2. Find the volume of the solid that is bounded by the cylinder $x^2 + z^2 = 4$ and the planes $y = 0$ and $y + z = 3$.

Problem 3. Give five other iterated integrals that are equal to $\displaystyle\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x,y,z)\,dz\,dx\,dy$.

Problem 4. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 2xy, x^2\rangle$ and C is the upper half of the semicircle of radius 1 from $(1,2)$ to $(3,2)$.

Problem 5. Evaluate $\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$ where C is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,3)$.

Problem 6. Evaluate $\displaystyle\int_C \left(\frac{y^2}{2} - \cos x\right) dx + \left(\frac{x^2}{2} + xy + \cos y\right) dy$ where C is the part of the circle $x^2 + y^2 = 1$ above the line $y = x$, oriented counterclockwise. [*Suggestion:* Use Green's Theorem to replace C with a much simpler curve.]