Math 2321 Spring 2009

Calculus III

Final Exam

Saturday, May 9

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** Except on multiple choice questions, you must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ________________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Points	4	4	4	4	4	4	4	8
Score								

Problem	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Points	8	12	12	8	8	8	8	10
Score								

Total:______________________

1. If $\mathbf{F}(x, y, z) = (2xz + y^2)\,\mathbf{i} + 2xy\,\mathbf{j} + (x^2 + 3z^2)\,\mathbf{k}$ and C is any curve from $(0, 1, -1)$ to $(1, 2, 1)$ then $\int_C \mathbf{F} \cdot d\mathbf{r}$ equals:

a. -7

b. 7

c. 0

d. It depends on C

2. Find $\nabla \times \mathbf{F}$ if $\mathbf{F}(x, y, z) = xyz\,\mathbf{i} - x^2y\,\mathbf{k}$.

a. $-2xy - xz$

b. $yz\,\mathbf{i}$

c. $-x^2\,\mathbf{i} + 3xy\,\mathbf{j} - xz\,\mathbf{k}$

d. $-x^2\,\mathbf{i} + xy\,\mathbf{j} - xz\,\mathbf{k}$

3. Suppose that $V(x, y, z) = 5x^2 - 3xy + xyz$. What is the rate of change of V at $(3, 4, 5)$ in the direction of $\mathbf{i} + \mathbf{j} - \mathbf{k}$?

a. 38

b. $\dfrac{32}{\sqrt{3}}$

c. $\dfrac{38}{\sqrt{3}}$

d. 32

4. The cylinder $x^2 + y^2 = 2$ intersects the plane $3x - 3y + z = 7$ in an ellipse. Find a vector equation for the line tangent to this ellipse at the point $(1, 1, 7)$.

a. $\mathbf{r}(t) = \langle 1 + t, 1 - t, 7 - 6t \rangle$

b. $\mathbf{r}(t) = \langle 1 + 2t, 1 + 2t, 7 \rangle$

c. $\mathbf{r}(t) = \langle 1 + 3t, 1 - 3t, 7 + t \rangle$

d. $\mathbf{r}(t) = \langle 1 + t, -1 + t, -6 + 7t \rangle$

5. If $u = 3x^2 + 2xy$, and x and y are both functions of s and t then $\dfrac{\partial u}{\partial s}$ is:

a. $(8x + 2y)\left(\dfrac{\partial x}{\partial s} + \dfrac{\partial y}{\partial s}\right)$

b. $(8x + 2y)\left(\dfrac{\partial x}{\partial s} + \dfrac{\partial y}{\partial t}\right)$

c. $2x\dfrac{\partial x}{\partial s} + (6x + 2y)\dfrac{\partial y}{\partial s}$

d. $(6x + 2y)\dfrac{\partial x}{\partial s} + 2x\dfrac{\partial y}{\partial s}$

6. The flux of $\mathbf{F}(x, y, z) = \langle y, x + y, z - y\rangle$ through the boundary of the cylinder $y^2 + z^2 = 3$, $-1 \leq x \leq 1$ is:

a. 18π

b. 12π

c. 36π

d. 6π

7. What is the Jacobian of the transformation $x = u^3 - 3uv^2$, $y = 3u^2v - v^3$?

a. $(3u^2 - 3v^2)\,\mathbf{i} + (3u^2 - 3v^2)\,\mathbf{j}$

b. $6u^2 - 6v^2$

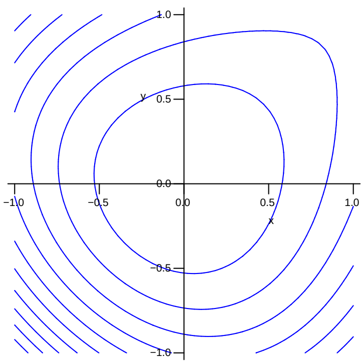
c. $9(u^2 + v^2)^2$

d. $9(u^2 + v^2)^2\,\mathbf{k}$

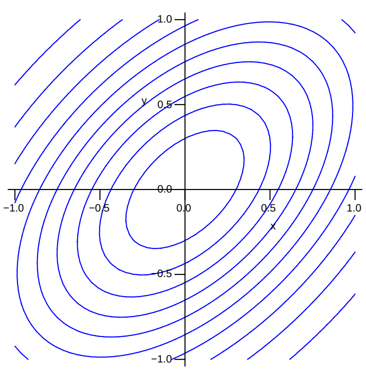
8. Match the function with its contour map and gradient vector field (shown on the next page).

	Contour map	Gradient field
$f(x, y) = x^2 - xy + y^2$		
$f(x, y) = e^{-x^2+xy-y^2}$		
$f(x, y) = \left(1 - \dfrac{(x+y)^2}{4}\right) \cos \pi(x - y)$		
$f(x, y) = (x - y)^2 - \dfrac{(x+y)^3}{3} + (x + y)^2$		

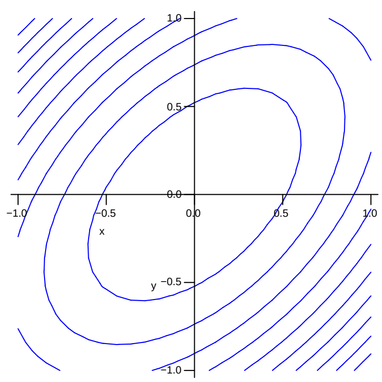
(A)



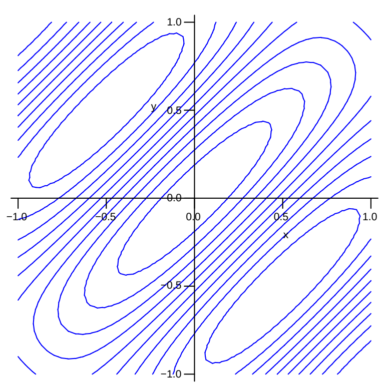
(B)



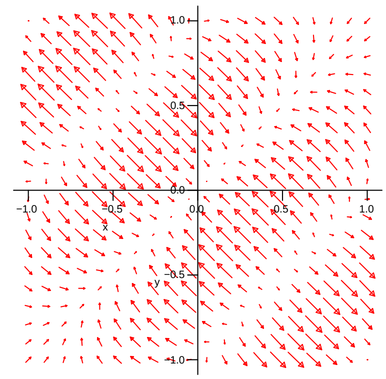
(C)



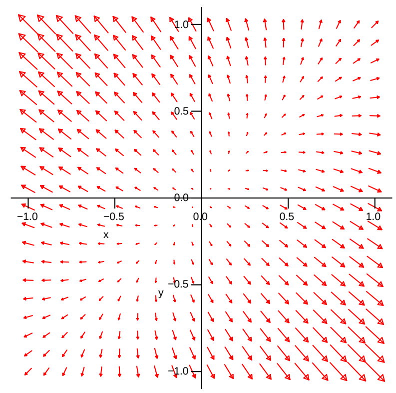
(D)



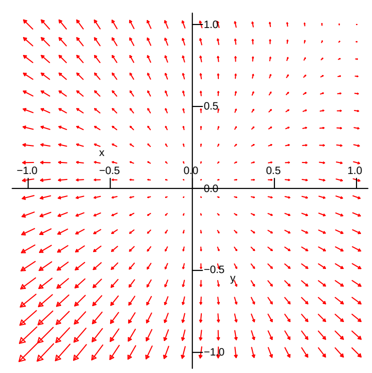
(I)



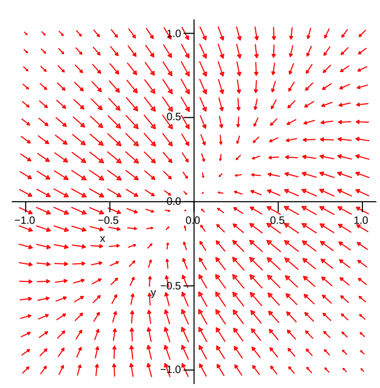
(II)



(III)



(IV)



9. Let $\mathbf{F}(x, y, z) = x^2\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$. If C is the line segment from $(0, 1, 1)$ to $(1, 1, 0)$, evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

10. Find the flux of $\mathbf{F}(x, y, z) = z\,\mathbf{j} + y\,\mathbf{k}$ through the part of the plane $3x + y + z = 3$ in the first octant, oriented upward.

11. Let S be the part of the graph of $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$. Compute $\iint_S x^2 + y^2 \, dS$.

12. Find the absolute maximum and minimum values of $f(x, y) = 3x - 4y$ on the circle of radius 10 centered at the origin.

13. Evaluate

$$\int_C (\ln x + y)\, dx - x^2\, dy$$

where C is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,2)$.

14. If $\mathbf{F}(x,y,z) = \langle y, z, x^2 - 3z\rangle$, compute $\displaystyle\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ for the surface S consisting of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, together with the cylinder $x^2 + y^2 = 1$, $-2 \leq z \leq 0$, oriented outward.

15. Find and classify the critical points of $f(x, y) = x^3 - 12xy + 8y^3$.

16.

a. If E is a solid region with boundary surface S, oriented outward, and $\nabla \cdot \mathbf{F} = 1$ show that

$$\mathrm{Vol}(E) = \iint_S \mathbf{F} \cdot d\mathbf{S}.$$

b. Let $0 < b < a$. Use part (a) to compute the volume enclosed by the torus

$$\begin{aligned} x &= (a + b\cos\alpha)\cos\theta, \\ y &= (a + b\cos\alpha)\sin\theta, \\ z &= b\sin\alpha, \end{aligned}$$

$0 \le \alpha \le 2\pi$, $0 \le \theta \le 2\pi$. [*Hint:* Try taking $\mathbf{F}(x, y, z) = z\mathbf{k}$.]