Introduction to Abstract Mathematics
Spring 2008
Review Problems for Exam 1

Review Problem 1: On each planet in a planetary system consisting of an odd number of planets there is an astronomer observing the nearest planet. The distances between each pair of planets are all different. Prove that at least one planet is not observed by any astronomer.

Review Problem 2: Given a function $f : X \to Y$, for $A, B \subseteq X$ we can define the set $f(A) = \{f(x) | x \in A\} \subseteq Y$ and similarly for B.

a. Show that $f(A \cup B) = f(A) \cup f(B)$.

b. Show that $f(A \cap B) \subseteq f(A) \cap f(B)$.

c. Find an example where $f(A \cap B) \neq f(A) \cap f(B)$.

Review Problem 3: Given $f : A \to B$, we can define a relation $\sim$ on X via $x \sim y$ if $f(x) = f(y)$.

a. Show that $\sim$ is an equivalence relation.

b. Describe $[x]$ and show that f is injective if and only if $[x] = \{x\}$, i.e, $[x]$ is a singleton.

Review Problem 4: Let X and Y be sets. Prove the following statements.

a. The function $f : X \to Y$ is an injection if and only if there exists a function $g : Y \to X$ so that $g \circ f = I_X$.

b. The function $g : Y \to X$ is a surjection if and only if there exists a function $f : X \to Y$ so that $g \circ f = I_X$.

Review Problem 5: Find negations of the statements "P and Q," "P or Q," "If P, then Q," and

Review Problem 6: Consider the function $h : X \to Y$ defined by $h(x) = x^2 + 1$.

a. Show that if $X = Y = \mathbb{R}$, then h is neither injective nor surjective.

b. Show that If $X = R$ and $Y = [1, \infty)$, then h is surjective but not injective.

c. Show that if $X = [0, \infty)$ and $Y = [1, \infty)$, then h is bijective.

Review Problem 7: Let $n \in \mathbb{N}$ and suppose X is a set with n elements. Find, with proof, the number of bijections from X to itself.

Review Problem 8: Show that for any two sets A and B, $A \cup B = (A \cap B) \cup (A - B) \cup (B - A)$.