Math 2326 - Introduction to Abstract Mathematics
Assignment 11 - Due Wednesday, February 13

Problem 41:

Let

$$R = \left\{(x, y) \in \mathbb{R}^2 \,|\, x = yr^2 \text{ for some } r \in \mathbb{R} - \{0\}\right\}.$$

a. Prove that R is an equivalence relation on $\mathbb{R}$.

b. If $x \in \mathbb{R}$, find a simple description of $\{y \in \mathbb{R} \,|\, (x, y) \in R\}$. [*Hint:* You should consider separately the cases $x < 0, x = 0$ and $x > 0$.]

c. If $\mathbb{R}$ is replaced with $\mathbb{Z}$ everywhere in the definition of R, is R an equivalence relation on $\mathbb{Z}$?

Problem 42:

Define a relation $\sim$ on $\mathbb{Z} \times \mathbb{N}$ by $(a, b) \sim (c, d)$ if and only if $ad - bc = 0$. Prove, using only properties of integers (i.e. without introducing fractions), that this is an equivalence relation.