Math 2326 - Introduction to Abstract Mathematics
Assignment 12 - Due Friday, February 15

Problem 43:

Let $m \in \mathbb{N}$ be a *fixed* positive integer. Given $x, y \in \mathbb{Z}$ we say $x \equiv y \pmod{m}$ if and only if m divides $x - y$. Show that $\equiv$ is an equivalence relation on $\mathbb{Z}$. This equivalence is called *congruence modulo* m.

Problem 44:

Let $A = \mathbb{R} \times \mathbb{R} - \{(0,0)\}$. Given $(a,b),(c,d) \in A$, define $(a,b) \sim (c,d)$ to mean $(a,b) = (rc, rd)$ for some $r \in \mathbb{R}$ with $r \neq 0$.

a. Prove that $\sim$ is an equivalence relation on A.

b. Viewing $\mathbb{R} \times \mathbb{R}$ as the set of points in a plane, describe the equivalence class $[(a,b)]$ geometrically.

c. Given $(a,b) \in A$ show that $[(a,b)]$ belongs to $\{[(x,1)] \,|\, x \in \mathbb{R}\} \cup \{[(1,0)]\}$.

Problem 45:

Prove that the relation

$$R = \left\{(x,y) \in \mathbb{Z}^2 \middle| |x - y| \leq 2\right\}$$

is *not* an equivalence relation.

Problem 46:

If $A = \{1, 2, 3, \ldots, 9, 10\}$ and $\mathcal{S} = \{\{1,3,5\},\{2,8\},\{4,6,9,10\},\{7\}\}$, find an equivalence relation on A whose set of equivalence classes is exactly $\mathcal{S}$.