Math 2326 - Introduction to Abstract Mathematics
Assignment 14 - Due Wednesday, February 20

Problem 53:

Let $f : X \to Y$ be a function and suppose that the functions $g : Y \to X$ and $h : Y \to X$ are both inverses for f, i.e. that $f \circ g = f \circ h = I_Y$ and $g \circ f = h \circ f = I_X$. Prove that $g = h$.

Problem 54:

Let $f : X \to Y$ and $g : Y \to Z$ be functions.

a. Prove that if f and g are both injective then so is $g \circ f$.

b. Prove that if f and g are both surjective then so is $g \circ f$.

c. If $g \circ f$ is injective do either of f or g have to be injective? Prove your answer.

d. If $g \circ f$ is surjective do either of f or g have to be surjective? Prove your answer.

Problem 55:

Prove that the function $f : \mathbb{R} \setminus \{2\} \to \mathbb{R}$ defined by $f(x) = x/(x-2)$ is *not* a bijection. Find a set $Y \subset \mathbb{R}$ so that the function $\widehat{f} : \mathbb{R} \setminus \{2\} \to Y$ given by the same formula *is* a bijection, and find $\widehat{f}^{-1}$.