Math 2326 - Introduction to Abstract Mathematics
Assignment 19 - Due Wednesday, March 5

Problem 68: Suppose that G is a group and that $g \in G$. Show that for all $n \in \mathbb{N}$, $(g^n)^{-1} = (g^{-1})^n$.

Problem 69: Let n be an integer with $n \geq 2$ and consider the set

$$U(n) = \{d \mid \gcd(d, n) = 1 \text{ and } 1 \leq d < n\}.$$

For example $U(12) = \{1, 5, 7, 11\}$. If we define multiplication in $U(n)$ as multiplication mod n, then $U(n)$ will be a group for all $n \geq 2$. For example, in $U(12)$, $5 \cdot 7 = 11$ since $35 \equiv 11 \pmod{12}$.

a. List the elements of $U(14)$ and state the order of $U(14)$.

b. Make a Cayley table for $U(14)$.

c. Find the orders of each element in $U(14)$.

Problem 70: Consider the group $\mathbb{Z}_{12}$.

a. What is the order of $\mathbb{Z}_{12}$?

b. Find the orders of each element in $\mathbb{Z}_{12}$.

c. When looking at the elements that have order 12, what do they have in common relative to the order of the group.

d. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Show that $\mathbb{Z}_{12} = \{5k \mid k \in S\}$. Here we take $5k = \overbrace{5 + 5 + \cdots + 5}^{\text{k-times}}$ mod 12.