Math 2326 - Introduction to Abstract Mathematics
Assignment 24 - Due Wednesday, March 26

Problem 83: Suppose that G, H, and K are groups and further suppose that $\phi : G \to H$ and $\psi : H \to K$ are isomorphisms, that is, $G \approx H$ and $H \approx K$. Show that G is isomorphic to K, that is, $G \approx K$.

Problem 84: Suppose that $(G_1, *_1)$ is a group with identity element e_1 and $(G_2, *_2)$ is a group with identity element e_2. If $\phi : G_1 \to G_2$ is an isomorphism, prove the following properties.

a. $\phi(e_1) = e_2$.

b. x and y commute in G_1 if and only if $\phi(x)$ and $\phi(y)$ commute in G_2.

c. For every $x \in G_1$ and every integer n, $\phi(x^n) = [\phi(x)]^n$.

Problem 85: Show that if G is a cyclic group of infinite order, then $G \approx \mathbb{Z}$.