Math 2326 - Introduction to Abstract Mathematics
Assignment 31 - Due Monday, April 14

Problem 102: Considering the definition of *supremum* of a subset of $\mathbb{R}$, formulate the definition of a *lower bound* of a set and the *infimum* in a similar fashion.

Problem 103: A set is *bounded* if it is bounded above and bounded below. Show that $S \subseteq \mathbb{R}$ is bounded if and only if there is and interval $I \subseteq \mathbb{R}$ such that $S \subseteq I$.

Problem 104: Consider the characterization of supremum stated in class:

Theorem: Let $A \subset \mathbb{R}$ and $s \in \mathbb{R}$. We have $s = \sup A$ if and only if s is an upper bound of A and for every $\varepsilon > 0$, there is $a \in A$ such that $s - \varepsilon < a$.

State a similar theorem characterizing the infimum of a set.

Problem 105: Determine if the supremum and infimum of each subset of $\mathbb{R}$ exists. If so, determine it. (You do not need to justify your answer.)

a. $(0, 1)$.

b. $\left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}$.

c. $\left\{ \frac{n}{n+1} \mid n \in \mathbb{N} \right\}$.

d. $\left\{ \frac{1}{m} + \frac{1}{n} \mid m, n \in \mathbb{N} \right\}$.

e. $\{ x \in \mathbb{Q} \mid x > 0 \}$.

f. $\{ x \in \mathbb{R} \mid x > 0 \}$.