Math 2326 - Introduction to Abstract Mathematics
Assignment 32 - Due Wednesday, April 16

Recall the following results from lecture and done in the homework characterizing the supremum and infimum of a set.

Theorem 1: Let $A \subset \mathbb{R}$ and $s \in \mathbb{R}$. We have $s = \sup A$ if and only if s is an upper bound of A and for every $\varepsilon > 0$, there is $a \in A$ such that $s - \varepsilon < a$.

Theorem 2: Let $A \subset \mathbb{R}$ and $\ell \in \mathbb{R}$. We have $\ell = \inf A$ if and only if ℓ is a lower bound of A and for every $\varepsilon > 0$, there is $a \in A$ such that $\ell + \varepsilon > a$.

Problem 106: Prove that if b is a lower bound for $B \subseteq \mathbb{R}$ and b is also an element of B, then it must be that $b = \inf B$.

Problem 107: Determine *with proof* if the supremum and infimum of each subset of $\mathbb{R}$ exists.

a. $(0, 1)$.

b. $\left\{\frac{n}{2n+1} \mid n \in \mathbb{N}\right\}$.

c. $\left\{n - \frac{1}{n} \mid n \in \mathbb{N}\right\}$.