Math 2326 - Introduction to Abstract Mathematics
Assignment 33 - Due Friday, April 18

Problem 108: Let $p \in \mathbb{Q}$, $p \neq 0$, and $x \in \mathbb{R} - \mathbb{Q}$. Show that px, $p + x$ are irrational. Give an example of x, y irrational such that $x + y$ and xy are rational.

Problem 109: Show that there is no rational number whose square is 2.

Problem 110: Finish the proof that $\alpha = \sup \{x \in \mathbb{R} |\ x \geq 0, x^2 < 2\} = \sqrt{2}$ by showing that the case $\alpha^2 > 2$ leads to a contradiction.

Problem 111: Let $A \subseteq \mathbb{R}$ be a nonempty bounded set. Given $c > 0$, consider the set $c \cdot A = \{cx |\ x \in A\}$. Show that $c \cdot A$ is bounded, and in fact, $\sup(c \cdot A) = c \sup A$ and $\inf(c \cdot A) = c \inf A$.