Math 2326 - Introduction to Abstract Mathematics
Assignment 34 - Due Monday, April 21

Problem 112: Finish the proof of the Nested Interval Property as indicated in the classroom.

Theorem: Let $I_n = [a_n, b_n] \subseteq \mathbb{R}$ be a nonempty interval for each $n \in \mathbb{N}$. Assume that $I_{n+1} \subseteq I_n$. Then $\bigcap_{n=1}^{\infty} I_n \neq \emptyset$, that is, there is $x \in \mathbb{R}$ so that $x \in I_n$, for all $n \in \mathbb{N}$. Furthermore, $\bigcap_{n=1}^{\infty} I_n = [a, b]$, where $a = \sup\{a_n\}$ and $b = \inf\{b_n\}$.

Problem 113: Prove the following:

Lemma: Given $I = [a, b] \neq \emptyset$ and $x \in \mathbb{R}$, there exists $J = [c, d] \neq \emptyset$ with $x \notin J$ and $J \subseteq I$.

Problem 114: Finish the inductive step in the proof that $\mathbb{R}$ is uncountable given during the lecture.