Math 2326 - Introduction to Abstract Mathematics
Assignment 4 - Due Monday, January 28

In Problems 15 through 17, prove the given statement.

Problem 15:

There exists $n \in \mathbb{Z}$ so that $n^2 + 2n + 1 = 0$.

Problem 16:

There exist infinitely many $m \in \mathbb{Z}$ so that 5 divides $3m - 1$.

Problem 17:

For all $r, s \in \mathbb{Q}$, $r + s \in \mathbb{Q}$ and $rs \in \mathbb{Q}$.

Problem 18: Consider the following proposition and its proof.

Proposition. For all $a, b \in \mathbb{N}$, $a^2 \neq 2b^2$.

Proof. Assume that there exist $a, b \in \mathbb{N}$ so that $a^2 = 2b^2$. Let m be the number of times 2 occurs in the prime factorization of a and let n be the number of times 2 occurs in the prime factorization of b. Then $2m$ is the number of times 2 occurs in the prime factorization of a^2 and $2n + 1$ is the number of times 2 occurs in the prime factorization of $2b^2$. Since $a^2 = 2b^2$ and prime factorizations are unique, it must be the case that $2m = 2n + 1$. We conclude that $2m$ is both even and odd, which is a contradiction. Therefore, for all $a, b \in \mathbb{N}$, $a^2 \neq 2b^2$. □

a. Find statements P and Q so that the proposition can be expressed in the form "If P then Q."

b. Express the assumption made at the beginning of the proof in terms of P and Q.

c. Let R be the statement "$2m$ is even." Express the conclusion of the proof in terms of only R and logical connectives.

d. Explain why this is a valid proof of the proposition.